Typos, Errors and Additional Notes

(16 April 2013)

Chapter 1.

- p. 2, in definition of a *proper function*: specify the domain is not empty.
- p. 4, Figure 1.1 Caption: function is from Chapter 7 not Chapter 6.
- p. 4, Theorem 1.2.2 last part of (c) should say: in *infinite* dimensions lower-semicontinuity is not automatic.
- p. 8, definition of distance function the inf should be over $s \in S$: $\inf_{s \in S} \|x - s\|$
- p. 9, Example 1.3.6: (c) $\log f$ is *a* convex function.
- p. 9, line −17: $\beta(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$
- p. 10, Example 1.39 should say suppose g is concave (which implies log g is concave)

Chapter 2.

- p. 21, line 2: whose epigraph is the closure *of the epigraph* of f
- p. 21: *Convex combinations* should have been introduced with convex hulls and the basic characterization of convex hulls using convex combinations should have been given in Section 2.1. Details are available at
  http://faculty.lasierra.edu/∼jvanderw/ConvexFunctions/Notes/convcomb.pdf
- p. 27, Proof of Fact 2.1.16: $f(t\bar{x} + d)$ should be $f(\bar{x} + td)$
- p. 32, Hint for Exercise 2.1.24: $x \log(n) \le (n + 1 + x) - g(n + 1) \le x \log(n + 1)$ should be $x \log(n) \le g(n+1+x) - g(n+1) \le x \log(n+1+x)$. And right-most side of first displayed inequality should be $x \log\left(1 + \frac{1}{n}\right)$ in place of $\log\left(1 + \frac{1}{n}\right)$
- p. 35, further details for the argument suggested in the second last sentence of the proof of Theorem 2.2.1 can be found at
  http://faculty.lasierra.edu/∼jvanderw/ConvexFunctions/Notes/Thm221.pdf
- p. 42, Exercise 2.2.14: Rado's result: "$k(A_k - G_k) \ge (k - 1)(A_{k-1}/G_{k-1})$" should be replaced by "$k(A_k - G_k) \ge (k - 1)(A_{k-1} - G_{k-1})$"
- p. 47, line −7: $p \ge d$ (should match theorem statement)
- p. 51, line 17: $\inf_{x \in E}\{($ the '(' is missing
- p. 52, Exercise 2.3.14(f): note the convex function $f : E \to \mathbb{R}$ is automatically continuous by Theorem 2.1.12
- p. 53, 2.3.15(c): $f : E \to \mathbb{R}$ (just for consistency, though valid in any normed space X)
- p. 53, 2.3.17(a): $f : E \to (-\infty, +\infty]$ (just for consistency, though valid on any normed space X)

- p. 54, line −6: should be $\frac{1}{2}d_S^2$ on left-hand side in displayed equation
- p. 55, Theorem 2.3.7: The definition of h in the hint should be $h(x) := \inf_{\mathbb{R}} \ldots$
- p. 56, Exercise 2.2.23: Replace ϕ_0 with ϕ_1 in first constraint: $\int_T \phi_1(x(t), t)\, \mu(\mathrm{d}t) = b_1$
- p. 58, Exercise 2.3.27, in the definition of the polar function replace $(k_C)^\circ(x) :=$ with $k_C^\circ(x^*) :=$
- p. 60, Exercise 2.3.28(d): missing period at end of statement.
- p. 60, Exercise 2.3.29, the definition of D_f should be: $f(x) - f(y) - \langle f'(y), x - y\rangle$
- p. 65, $\phi \neq 0$ in Corollary 2.4.3.
- p. 66, Thm 2.4.6: Suppose $\dim E > 0$
- p. 67, Thm 2.4.7: Note a stronger result is available, namely the separation holds if and only if $\mathrm{ri} C_1 \cap \mathrm{ri} C_2 = \emptyset$. See Theorem 11.3 in Rockafellar's book *Convex Analysis* for this.
- p. 69, line 9: should be $\alpha(x) := \langle A^*\phi, x\rangle + r$
- p. 70: in (2.4.12): $\sum_{i=0}^m \lambda_i x_i$ (first sum should also be from $i = 0$ to $i = m$).
- p. 70: i is missing in (2.4.13): for $i = 0, 1, 2, \ldots, m$, $x \in E$.
- p. 81, Hint to Exercise 2.5.2: $G_{n,m} := \left\{ x \in U : \sup_{\|h\| \leq \frac{1}{m}} f(x+h) + f(x-h) - 2f(x) < \frac{1}{mn} \right\}$.
- p. 82, line −6: See [95, Chapter 6] *for* a
- Section 2.6: functions are tacitly assumed closed, e.g. Proposition 2.6.3
- p. 84, line −3: as $t \to 0^+$?
- p. 85, line −11: $h \in W$ (not $w \in W$)
- p. 85, Proposition 2.6.3: the function $\Delta_t^2 f(x)$ is closed nonnegative and convex
- p. 86, Theorem 2.6.4: *equivealently*, *generalied* should be *equivalently*, *generalized*
- p. 88, line −4, Exercise 2.6.4(c) should read: $\lim_{t \to 0} \frac{\phi_t - \nabla f(x)}{t} = Ah$ where $\phi \in \partial f(x + th)$.
- p. 91, Hint to 2.6.13, second last line should read: there is a continuous strictly increasing function g such that $g'(t) = \infty$ for all $t \in G$

Chapter 3.

- p. 99, the third line of Section 3.2: $\mathrm{Diag} : \mathbb{R}^n$
- p. 118, line 3 of proof of Theorem 3.5.3: should refer to Exercise 3.5.12
- p. 121, Exercise 3.5.7, the first line of Hint: $f(x) - f(x_0) + \epsilon$

Chapter 4.

- p. 131, Proposition 4.1.9, last line: $\bar{x} \in \operatorname{cont} f$
- p. 132, Corollary 4.1.12: $C \subset X \ldots \phi \in X^*$ (note automatically $\phi \neq 0$)
- p. 133, Remark 4.1.16(b): $\phi(x) = 1$;
- p. 136, Corollary 4.1.20: note (automatically) $\phi \neq 0$
- p. 136, Theorem 4.1.22: note (automatically) $x_0 \neq 0$
- p. 133, in the sandwich theorem: f and g can be extended real-valued function*s*
- p. 134, line −10: $+\frac{1}{2^i}S+$ (S is missing)
- p. 135, line 7: definitions of a and b should be interchanged
- p. 135, line −3: should be $g(Tx)$ not $g(Ax)$.
- p. 136, Proposition 4.1.23, line −3: one need only assume $x_0^* \in \operatorname{dom} f$, and on p. 137, line 5, it suffices to have $x_0^* \in \operatorname{dom} f$.
- p. 137, line 7: $\langle x_0, x_0^* - \alpha\rangle$ should be $\langle x_0, x_0^*\rangle - \alpha$
- p. 138,: In Hint for 4.1.2(b)(ii) "$f(x) = 0$ for all x in a dense convex subset …" should be replaced with something like "$f(x) = \Lambda(x)$" for all x in a dense convex subset of B_X where Λ is an appropriately chosen discontinuous linear functional …"
- p. 142, Exercise 4.1.21(a): $\Lambda : c_{00} \to \mathbb{R}$ (Λ will be extended to c_0 in (b))
- p. 146, Exercise 4.1.46(c): $f^*(x) = \|x\|^2/2$ if $x \in \overline{\operatorname{conv}} F$ and $f^*(x) = +\infty$ otherwise.
- p. 151, line 15: norm dense in S_X
- p. 154, Cor. 4.2.12: U is an open convex set
- p. 157, proof of Proposition 4.2.16: there are subtleties in the directional modification of the proof of Proposition 4.2.14 when f is not Lipschitz. Full details of the proof of Proposition 4.2.16 are at
  http://faculty.lasierra.edu/∼jvanderw/ConvexFunctions/Notes/Prop4216.pdf
- p. 160, line −2: strict *for distinct* $x, y \in U$.
- p. 167, line 3: *Then* there is
- p. 168, Hint to Exercise 4.3.6: specify $x_0 \in \operatorname{bnd} C$.
- p. 171, Proposition 4.4.1(c) should read: If $f \leq g$, then $f^* \geq g^*$.
- p. 172, Proposition 4.4.2(a) should include assumption f is lsc at some point of its domain (to apply epi-separation theorem)
- p. 173, 2nd line of proof of Fact 4.4.4: $\{u \in X^{**} : f^{**}(u) \leq K\}$ (capital K)
- p. 188, Fact 4.5.3, In particular statement specify nonempty: every *nonempty* closed convex

- p. 190, Theorem 4.5.7(c) should read: d_C^2 is Gâteaux differentiable and C is proximal. For some further information related to distance functions and differentiability see http://faculty.lasierra.edu/~jvanderw/ConvexFunctions/Notes/distancefun.pdf
- p. 191, Exercise 4.5.4 should additionally assume: A is proximal
- p. 193, Exercise 4.5.9: in definition of approximately convex the ball D must be disjoint from C
- p. 194, Exercise 4.5.10, definition of a sun: "for $x \in S$" should be "for any x in the Hilbert space"
- p. 202, Theorem 4.6.13: If $\nabla^2 f(\bar{x})(h,h) > 0$ for all $\bar{x} \in U$, $h \in S_X$, then f is strictly convex.
- p. 204, Remark 4.6.17(d): mention that C has empty interior
- p. 205, Exercise 4.6.4(b): end question with "?"
- p. 205, Exercise 4.6.6, specify $x \neq y$ in definition of strictly convex norm: whenever $\|x\| = \|y\|$ and $x \neq y$

Chapter 5.

- tacitly uses $\delta(\cdot)$ or $\delta_{\|\cdot\|}(\cdot)$ in place of $\delta_X(\cdot)$.
- p. 217, line −2: forevery
- p. 221, last line of proof of Theorem 5.1.32 should read: of weaker results (delete 'a')
- p. 236, Hint to Exercise 5.2.6(b): $f^*(ne_n) - \langle 0, ne_n \rangle = \frac{n}{2^n}$
- p. 236, second line of Hint to Exercise 5.2.6: now implies $(f-\phi)(u) \geq (f-\phi)(x) + n\delta$ (should not be $>$)
- p. 241: It is worth noting that each condition in Theorem 5.3.7(b) is equivalent to f^* being *strongly rotund* as introduced in Section 6.4. For details see http://faculty.lasierra.edu/~jvanderw/ConvexFunctions/Notes/Thm537.pdf
- p. 244: $\delta_f : [0,\infty) \to [0,\infty]$

Chapter 6.

- p. 276, line −7: replace $T(S) = \bigcup_{x \in A} T(x)$ with $T(S) = \bigcup_{x \in S} T(x)$
- p. 279, last line of proof of Theorem 6.1.5 should read: $\ldots v_0 \in \partial f(x_0)$ as desired.
- p. 280 Thm 6.1.7 and its proof T_U should be $T|_U$
- p. 283, Exercise 6.1.8 should say *Hint* and not *Proof*.

Chapter 7.

- p. 339: Lemma 7.2.2: $x \in \operatorname{bndy} \operatorname{dom} \partial f$
- p. 340, first line: $x \in \operatorname{bndy} \operatorname{dom} \partial f$
- p. 347, Third line of Hint to Exercise 7.3.3(b): should be Lemma 7.3.1(a) not 7.3.1(i)
- p. 348, Exercise 7.3.7: extra)
- p. 351: Theorem 7.4.4 parenthetical statement should read: where we note $(A-x)^\circ$ is equal to the negative polar cone $(A-x)^-$

Chapter 9.

- p. 404, note f need not be closed and convex
- p. 404, second paragraph: the subdifferential of a *proper* convex lsc function on a Banach space is the typical ...
- p. 406, Corollary 9.1.5: f and g need to also be lsc
- p. 436, last line: $\langle y_n^*, y_n \rangle = -\langle j_n^*, y_n \rangle$
- p. 451: Theorem 9.7.2 should be for $x \in X$ or $\hat{x} \in X^{**}$.
- p. 451, proof of Theorem 9.7.2, line 6: right parenthesis should be before $\le$: $\max(\ldots) \le 0$
- p. 454: Exercise 9.7.9(d): ... of the *Hamel* basis to F.

Chapter 10.

- p. 460: 4th line of 10.1.1 *a* real-valued convex function*s*
- p. 477, line 9: in various forms *for* were given

Index.

- p. 512, line −18, left column: *characerization*

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