Theory and Applications of Convex and Non-convex Feasibility Problems

Laureate Prof. Jonathan Borwein with Matthew Tam
http://carma.newcastle.edu.au/DRmethods/paseky.html

Spring School on Variational Analysis VI
Paseky nad Jizerou, April 19–25, 2015

Last Revised: May 6, 2016

Spring School on Variational Analysis 2015

For Spring School on Function Spaces and Lineability 2015, click here

What am I if I will not participate?
- Antoine de Saint-Exupery

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Dear Colleague,

Following a longstanding tradition, the Faculty of Mathematics and Physics, Charles University in Prague and the Academy of Sciences of the Czech Republic will organize the Spring School on Variational Analysis VI. The School will be held in Paseky nad Jizerou, in a chalet in the Krkonose Mountains, **April 19 - 25, 2015**.

The program will consist of series of lectures on

Variational Analysis

and its Applications

delivered by

Jonathan M. Borwein
The University of Newcastle, Australia
Theory and Applications of Convex and Non-convex Feasibility Problems

Marián Fabian
Academy of Sciences of the Czech Republic, Prague, Czech Republic
Separable Reductions and Rich Families in Theory of Fréchet Subdifferentials

Alexander Ioffe
Technion, Haifa, Israel
Variational Analysis and Optimization Theory

David Russell Luke
Georg-August-Universität Göttingen, Germany
Variational Methods in Numerical Analysis

The purpose of this meeting is to bring together researchers with common interest in the field. There will be opportunities for informal discussions. Graduate students and others beginning their mathematical career are encouraged to participate.

Introduction

A feasibility problem requests solution to the problem

$$\text{Find} \quad x \in \bigcap_{i=1}^{N} C_i,$$

where $C_1, C_2, \ldots C_N$ are closed sets lying in a Hilbert space $\mathcal{H}$.

We consider iterative methods based on the non-expansive properties of the metric projection operator

$$P_C(x) := \operatorname{argmin}_{c \in C} \|x - c\|$$

or reflection operator $R_C := 2P_C - I$ on a closed convex set C.

The two methods which we focus are on the method of alternating projections (MAP) and the Douglas–Rachford method (DR).

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Introduction

These methods work best when the projection on each set C_i is easy to describe or approximate. These methods are especially useful when the number of sets involved is large as the methods are fairly easy to parallelize.

The theory is pretty well understood when all sets are convex but much less clear in the non-convex case. But as we shall see application of this case has had may successes. So this is a fertile area for both pure and applied study.

The five hours of lectures will cover the following topics.

1. **Feasibility problems:** convex theory, nonexpansivity, Fejér monotonicity & convergence of MAP and variants.
2. **The Douglas–Rachford Method:** convex Douglas–Rachford iterations and variants.
3. **Non-convex Douglas Rachford iterations** and **iterative geometry**.
4. **Applications to completion problems**: an introduction & detailed case studies.

- Each lecture will contain closing commentary, open questions, and exercises.

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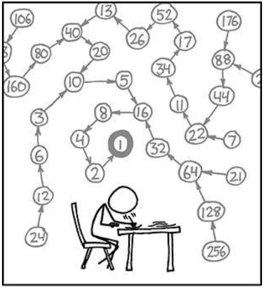
Motivation

The need to integrate and iterate real theory with real models for real applications:

- Good theoretical understanding
  - you can not use what you do not know
  - you can work inductively
- Careful modelling of applications
  - the model matters especially in the nonconvex case
  - moving to application specific refinements
- Good implementations
  - starting with 'general purpose agents'
  - moving to application specific refinements

Introduction



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Lectures are available online at:

http://carma.newcastle.edu.au/DRmethods/paseky.html