Neverending Fractions

An Introduction to Continued Fractions

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Preface

This book arose from many lectures the authors delivered independently at different locations to students of different levels.

'Theory' is a scientific name for 'story'. So, if the reader somehow feels uncomfortable about following a theory of continued fractions, he or she might be more content to read the story of *neverending* fractions.

The queen of mathematics – number theory – remains one of the most accessible parts of significant mathematical knowledge. Continued fractions form a classical area within number theory, and there are many textbooks and monographs devoted to them. Despite their classical nature, continued fractions remain a neverending research field, many of whose results are elementary enough to be explained to a wide audience of graduates, postgraduates and researchers, as well as teachers and even amateurs in mathematics. These are the people to whom this book is addressed.

After a standard introduction to continued fractions in the first three chapters, including generalisations such as continued fractions in function fields and irregular continued fractions, there are six 'topics' chapters. In these we give various amazing applications of the theory (irrationality proofs, generating series, combinatorics on words, Somos sequences, Diophantine equations and many other applications) to seemingly unrelated problems in number theory. The main feature that we would like to make apparent through this book is the naturalness of continued fractions and of their expected appearance in mathematics. The book is a combination of formal and informal styles. The aforementioned applications of continued fractions are, for the most part, not to be found in earlier books but only in scattered scientific articles and lectures.

We have included various remarks and exercises but have been sparing with the latter. In the topics chapters we do not always give full details. Needless to say, all topics can be followed up in the end notes for each chapter and through the references.

We would like to thank many colleagues for useful conversations during the development of this book, especially Mumtaz Hussain, Pieter Moree and James Wan. We are also deeply indebted to the copy-editor Susan Parkinson whose incisive and tireless work on the book has enhanced its appearance immeasurably.

Finally, Alf van der Poorten (1942–2010) died before this book could be brought to fruition. He was both a good friend and a fine colleague. We offer this book both in his memory and as a way of bringing to a more general audience some of his wonderful contributions to the area. Chapters 4, 5 and 6 originate in lectures Alf gave in the last few years of his life and, for matters of both taste and necessity, they are largely left as presentations in his unique and erudite style.

Alfred "Alf" Jacobus van der Poorten
(16 May 1942 in Amsterdam–9 October 2010 in Sydney)

A full mathematical biography of Alf is to be found in the 2013 volume dedicated to his memory [31].

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