

# Report on J. Borwein and B. Sims, "The Douglas-Rachford Algorithm in the Absence of Convexity"

The authors study the behaviour of the Douglas-Rachford algorithm for finding the intersection of two sets in the instance when one of two sets is a line, or a line segment, and the other set is a ring. For this example, the iterates of the Douglas Rachford algorithm can be written as steps in a difference equation, leading to an analysis of the the Douglas-Rachford algorithm as a dynamical system. Specific comments follow.

1. document was not compiled enough to synchronize references. so I am not certain that these are consistent or complete.
2. Spelling errors throughout (e.g. <sup>P</sup>Spiralling, waek, Spitting, Ratchford).
3. p3 l+13: It seems that an opportunity is lost to at least mention one large "pathology" of nonconvex projections by excluding the point  $x = 0$  from the discussion, i.e. single-valuedness of the projectors. Perhaps a very brief mention that by excluding the origin you are ensuring that the projector and reflector corresponding to the circle/sphere is single-valued? Also, by excluding the origin from the discussion, you are excluding all initial points whose iterates pass through the origin (item below).
4. p4. Remark 1: "divide-and-concurr", though not so named by its inventor, is due to Pierra. Citation [9] should be replaced by author = G. Pierra, title = Eclatement de contraintes en parallèle pour la minimisation d'une forme quadratique, journal = Lecture Notes in Computer Science, publisher = Springer Verlag, address = New York, year = 1976, volume = 41, pages = 200-218. Elser "repackaged" it in [9], but the idea is well established and common practice in the mathematics literature.
5. p5, Example 1: I do not really understand the point of this example. Averaged reflections were never under consideration.
6. p6, Fig.3: the points do not match the description - the very first move, if really a reflection across the line segment, would be in a direction northwest from the initial point with midpoint at the right endpoint of the linesegment. Similarly with the third move. It appears that the algorithm thinks it's working on a line, not a line segment.
7. p6, Fig.3: Why show alternating reflections? It is known that this does not converge even in the convex case. Averaging, or the addition of a Krasnoselski-Mann relaxation, is important for restoring firm nonexpansiveness of the fixed point mapping in the convex case.
8. p8, Theorem 1: should  $f$  be a mapping from  $N \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  instead of  $\mathbb{R}^m$ ?  $\checkmark$
9. p8, Theorem 1: if you use  $n$  for the dimension of the domain and range, please use a different index for the iterate (instead of  $n$ ).  $\checkmark$  used  $m$
10. p9. l+2,  $T_y^f(x)$ : This is the first time this notation is used, and it doesn't match  $T_{S,L}$  very well. Consider an alternative notation?

see comment after sign  
7 T R.B  
on P 3

delete

if this notation is ok but pick different  $T := T^f$

please write  $U$  for a mapping  $\otimes \vee \odot$ !