BLUE SKY, RED SUN

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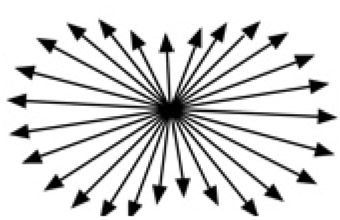
The blue colour of the sky and the reddening of the sun at sunrise and sunset are both due to the scattering of sun light by particles in the atmosphere, see figure 1(a). The particles involved are predominantly nitrogen and oxygen molecules, but also molecules of water and carbon dioxide, plus very fine dust. The diameters d of such particles are substantially smaller than the wavelengths λ of visible light, typically $d << \lambda/10$, leading to a type of scattering known as *Rayleigh scattering*; named after the British physicist Lord Rayleigh (John William Strutt, 3rd Baron Rayleigh, figure 1(b)).

Rayleigh was the first to investigate the phenomenon. His analysis, commenced in 1871 and extending over several years, showed that at a distance R from such a particle the intensity of light scattered at an angle $\theta \in [0, \pi]$ to the direction of the incident light (figure 2) as a fraction of the intensity of the incident light is,

$$I_s(\theta) \;=\; \frac{1+\cos^2\theta}{2R^2}\left(\frac{2\pi}{\lambda}\right)^4\left(\frac{n^2-1}{n^2+2}\right)^2\left(\frac{d}{2}\right)^6,$$

here n is the refractive index of the scattering media, which for N_2 at standard temperature and pressure is approximately 1.0003. (You can think of $I_s(\theta)$ as



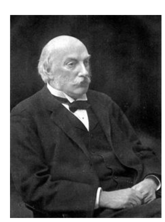


Figure 1: (a) Rayleigh scattering (b) Lord Rayleigh, 1842 — 1919

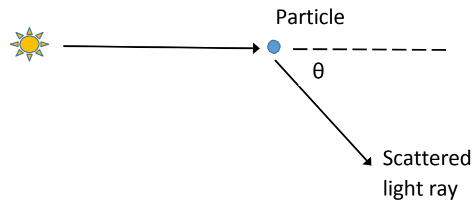


Figure 2: Scattering angle

similar to a probability density function for the distribution of the intensity of the scattered light as a function of the scattering angle.)

From this we observe that light scattered at right angles ($\theta = \pi/2$) to the direction of the incident light is half as intense as that scattered foreward or backward ($\theta = 0$ or π), observe the outer envelope of the rays in figure 1(a).

The *Rayleigh scattering cross section* σ_s of a particle is the overall portion of the light incident on it that is scattered. It is found by integrating the above expression for $I_s(\theta)$ over a unit sphere centred on the particle. Since the direction of the incident light is an axis of symmetry for the scattering, it is convenient to regard the sphere as a surface of revolution (see figure 3) to obtain

$$\sigma_s = \left(\frac{2\pi}{\lambda}\right)^4 \left(\frac{n^2-1}{n^2+2}\right)^2 \left(\frac{d}{2}\right)^6 \times \pi \int_0^\pi (1+\cos^2\theta)\sin\theta\, d\theta.$$

It is an easy exercise to show this leads to

$$\sigma_s = \frac{2\pi^5}{3}\left(\frac{d^6}{\lambda^4}\right)\left(\frac{n^2-1}{n^2+2}\right)^2.$$

Although refractive index varies with wavelength, the variation across the visible spectrum is slight, and the salient observation is that to a high degree of accuracy

$$\sigma_s \propto \frac{1}{\lambda^4}.$$

The blue potion of the spectrum is centred about a wavelength of $\lambda_B = 450$nm (nanometres), 4.5×10^{-7} metres, while the longer wavelength red light clusters around $\lambda_R = 625$nm, so approximately

$$\left(\frac{\lambda_R}{\lambda_B}\right)^4 = \left(\frac{625}{450}\right)^4 \approx 3.7$$

times more of the blue in sunlight is scattered than of the red. As a consequence, the scattered light, which appears to come from every point in the sky, has a

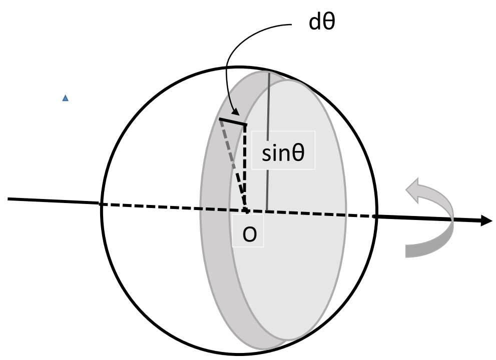


Figure 3: Integrating $I_s(\theta)$ over the unit sphere

distinctly blue tinge; hence **our blue sky**, and our nickname “the blue planet”, figure 5.

The significance of the Rayleigh scattering cross section lies in the observation that the proportion of the incident light that has been scattered before arriving at a particular destination equals $\sigma_s N$ where N is the number of intervening particles. So, the passage of direct sunlight through the atmosphere for a distance d reduces its intensity by an amount proportional to $\sigma_s d$. Again the inverse dependence of σ_s on the fourth power of the wavelength, together with some simple geometry, is important.

Denoting the radius of the solid earth by R ($\approx 6,378$km) and the effective depth of the atmosphere by h (≈ 15km; 80% of the atmosphere's mass is concentrated below this altitude) we see (refer to figure 4) that the thickness of atmosphere sunlight must traverse at sunrise or sunset to reach our observer at ground level is, neglecting the h^2 term, approximately

$$t \;=\; \sqrt{2Rh} \approx 437\text{km}.$$

Thus, the the direct sunlight reaching our observer at sunrise (or sunset) has lost $t/h \approx 30$ times more of its blue component than that arriving when the sun is overhead. Of course the same is also true for red light, but remember almost four times more blue light than red is filtered out through scattering. This greater diminution of blue due to the greater thickness of atmosphere traversed accounts for the familiar **copper-red colour of the rising or setting sun** compared to the disk's buttercup yellow appearance at other time of the day.

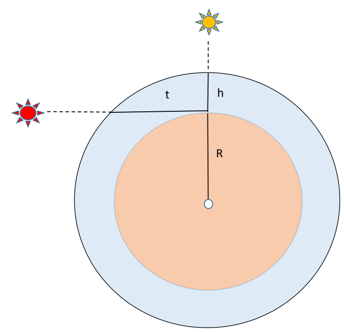


Figure 4: Thickness of atmosphere at sunrise/sunset

Figure 5: Left: Blue sky and red sunrise, Right: Blue earth

You might like to investigate the magnitude of this phenomenon when the elevation of the sun above the horizon is an angle other than 0^o or 90^o, and also the effect latitude might play.