AIMS-Stellenbosch Number Theory Conference 2017
Programme and Abstracts

16 – 20 January 2017

Time	Monday	Tuesday	Wednesday	Thursday	Friday
08h30-09h00			Patrick Rabarison		
09h00-09h30		Kenneth Hughes		Federico Pellarin	Richard Pink
09h30-10h00	Welcome		Luca Demangos		
10h00-10h30	Augustine Munagi	Moshe Jarden		Ahmad El-Guindy	Mihran Papikian
10h30-11h00			Coffee		
11h00-11h30	Coffee	Coffee	Jasbir Chahal	Coffee	Coffee
11h30-12h00	Fabien Pazuki	Aharon Razon		Dirk Basson	Alex Bamunoba
12h00-12h30			Lunch		
12h30-13h00	Lunch	Lunch		Lunch	Lunch
13h00-13h30			Travel to AIMS		
13h30-14h00					
14h00-14h30	Darlison Nyirenda	Gabino González Díez	Marc Hindry	Douglas Ulmer	Ambrus Pal
14h30-15h00	Keisuke Arai				
15h00-15h30	Coffee	Coffee	Coffee	Coffee	Coffee
15h30-16h00	Gareth Boxall	Cynthia Ramiharimanana	Beach	Sophie Marques	Sinnou David
16h00-16h30	Waleed Hassaan	Fabrizio Barroero			
16h30-17h00			Travel to Stellenbosch		
17h00-17h30					
19h00-22h00			Dinner at Katjiepiering		

Abstracts

Rational points on Shimura curves and counterexamples to the Hasse principle

Keisuke Arai

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In this talk, we give criteria of the non-existence of rational points on Shimura curves. From these criteria, we can produce an infinite family of counterexamples to the Hasse principle for Shimura curves, which are accounted for by the Manin obstruction.

Classical vs Carlitz Wieferich primes

Alex Bamunoba

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An interesting phenomenon in Number Theory is the deep analogy between number fields and global function fields. At the most accessible level, this is seen in the strong similarities between the ring $\mathbb{Z}$ of integers and the ring $\mathbb{F}_q[T]$ of polynomials over a finite field $\mathbb{F}_q$. Both rings are

Euclidean domains with finitely many units and infinitely many primes, the residue fields of which are finite. As a result, many basic results in Elementary Number Theory (over $\mathbb{Z}$) have analogues over $\mathbb{F}_q[T]$, such as the Euler-Fermat Theorem and Wilson's Theorem. At a deeper level, the analogy continues between on the one hand the multiplicative group $\mathbb{G}_{\mathrm{m}}$ over $\mathbb{Q}$ with its torsion points and the associated theory of cyclotomic fields and polynomials, and on the other hand the Carlitz module over $\mathbb{F}_q[T]$ and its associated torsion points and cyclotomic theory. In this talk, we shall utilise this analogy together with the theory of Carlitz modules to survey the Carlitzian analogue of some known results about classical Wieferich primes.

Unlikely Intersections in families of elliptic curves

Fabrizio Barroero

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Given n independent points on the Legendre family of elliptic curves of equation

$$Y^2 = X(X-1)(X-c)$$

with coordinates algebraic over $\mathbb{Q}(c)$, we will see that there are at most finitely many specializations of c such that two independent relations with integer coefficients hold between the n points on the specialized curve. Moreover, there are at most finitely many c such that the elliptic curve has complex multiplication and the n points are dependent over the larger endomorphism ring. This fits in the framework of the so-called Zilber-Pink conjectures on Unlikely Intersections.

A covolume form on the higher rank Drinfeld period domain

Dirk Basson

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When a matrix acts on a complex number by a fractional linear transformation, the imaginary part of the complex number transforms in a way that is analogous to a weight -2 modular form:

$$\Im(\gamma\tau) = \frac{\det\gamma \cdot \Im(\tau)}{|c\tau + d|^2}.$$

Gekeler showed that a similar formula holds for the Drinfeld upper half-plane, which is the space underlying Drinfeld modular forms of rank 2. However, a naïve translation of the "imaginary part" to the higher rank Drinfeld period domain seems not to allow such a formula.

In this talk I will elaborate on the formulas above and explain how the "imaginary part" can be replaced with a covolume form which does allow the generalization of such a formula to the higher rank Drinfeld period domain.

Nevanlinna theory and exponential polynomials

Gareth Boxall

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In the 1980s, Henson and Rubel used properties of the Nevanlinna characteristic to prove that if $f : \mathbb{C}^n \to \mathbb{C}$ is an exponential polynomial over $\mathbb{C}$ with no zeroes then there is an exponential polynomial g such that $f = e^g$. We use a slight refinement of this to show that if $X \subseteq \mathbb{C}^n$ is the zero set of an exponential polynomial equation and $\pi : \mathbb{C}^n \to \mathbb{C}^k$ is a projection, then $\pi(X)$ is a countable boolean combination of zero sets of exponential polynomial equations. This implies a special case of Zilber's quasiminimality conjecture for the complex exponential field.

Algebra, Arithmetic and Geometry - An Interplay

Jasbir Chahal

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I will discus some application of the arithmetic algebraic geometry to solving some number theoretic problems arising from classical geometry.

On Lehmer's Problem

Sinnou David

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We shall discuss the various aspects of Lehmer's problem from the classical statement by Lehmer to the more recent group variety set up. In the latter case, we shall concentrate on the semi-abelian case, characterize conjecturally obstructions and explain partial results in small dimension cases

Quantum j invariant and Hilbert 12th problem for real quadratic function fields – Joint work with T. M. Gendron

Luca Demangos

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We introduce the notion of quantum j invariant of quantum tori to generate the Hilbert class field of the relative ring of integers of real quadratic extensions of $\mathbb{F}_q(T)$.

Formulas and vanishing conditions for certain coefficients of Drinfeld-Goss Hecke eigenforms

Ahmad El-Guindy

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We obtain a closed form polynomial expression for certain coefficients of Drinfeld-Goss double-cuspidal modular forms which are eigenforms for the degree one Hecke operators with power eigenvalues, and we use those formulas to prove vanishing results for an infinite family of those coefficients.

On the action of the absolute Galois group on algebraic curves and surfaces

Gabino González Díez

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A combination of Hodge theory and Serre's GAGA principle implies that many topological invariants of complex projective varieties, such as Betti numbers and Chern classes, remain invariant under Galois action. Nevertheless, in 1964 Serre constructed examples of projective varieties X and Galois elements $\sigma \in \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ such that X and its conjugate variety X^σ have non-isomorphic fundamental groups, and so they are not homeomorphic. Further particular instances of this phenomenon have been discovered since then.

In this talk I will attempt to show that for every $\sigma \in \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ different from the identity or the complex conjugation there is a complex surface S defined over a number field such that the fundamental groups $\pi_1(S)$ and $\pi_1(S^\sigma)$ are non-isomorphic (although their profinite completions $\pi_1^{alg}(S)$ and $\pi_1^{alg}(S^\sigma)$ are). The surfaces S in question (Beauville surfaces) will arise as finite quotients of products of triangle curves.

This is joint work with Andrei Jaikin-Zapirain.

Weierstrass points of order three on smooth quartic curves

Waleed Hassaan

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Weierstrass points theory is one of classic topics in the study of algebraic curves. In this talk we study the geometric classification of the Weierstrass points of order three on smooth quartic curves and give all possible Weierstrass gap sequences of these points. Furthermore, we develop a technique to find the distribution of the Weierstrass points of order three on any smooth quartic curve.

Asymptotics for global fields and elliptic curves over these

Marc Hindry

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A global field is either a number field (a finite degree extension of the field of rationals) or the function field of an algebraic curve over a finite field (which can be viewed as a finite degree extension of the field $\mathbb{F}_p(T)$). Asymptotics means here the behaviour of "complicated" arithmetic quantities like the class number and regulator, when the "simple" quantity discriminant tends to infinity.

A typical example is the size of the smallest non trivial solution to Pell-Fermat equation $x^2 - dy^2 = 1$ when d goes to infinity.

The known answer is summarized by the classical Brauer-Siegel theorem, which I'll recall.

I will then discuss analogues for elliptic curves: here we are looking for the minimal size of generators of the group of rational points on a curve E with equation $y^2 = x^3 + ax + b$ The "simple" quantity is the (exponential) height $H(E)$ and the "complicated" quantity are the Néron-Tate regulator (which controls the size of generators) and the cardinality of the Shafarevich-Tate group.

Plane partitions and a possible "Third Way" in Number Theory.

Kenneth Hughes

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Traditionally number theory has been approached either via algebra or via harmonic analysis. Yet there is a third way – much neglected and under-developed – which seeks to develop number-theory using combinatorics. The big success here is the theory of partitions, which from RIEMANN, to Ken ONO, has linked elementary combinatorics to deep results on modular forms (ETA products). A test case for developing this theory further is provided by Major MacMAHON's theory of plane partitions, where elementary methods have had some success, but so far, the right approach to deeper analysis has eluded us. In this talk we report on recent work which tries to blend ideas from the MIT approach to combinatorics, ("multi-sets") with older analytic work inspired by BARNES's G-function.

Torsion on Abelian Varieties over Large Algebraic Extensions of Finitely Generated Extensions of $\mathbb{Q}$

Moshe Jarden

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Let K be a finitely generated extension of $\mathbb{Q}$ and A a non-zero abelian variety over K. Let $\tilde{K}$ be the algebraic closure of K and $\mathrm{Gal}(K) = \mathrm{Gal}(\tilde{K}/K)$ the absolute Galois group of K equipped with its Haar measure. For each $\sigma \in \mathrm{Gal}(K)$ let $\tilde{K}(\sigma)$ be the fixed of σ field in $\tilde{K}$. We prove that for almost all $\sigma \in \mathrm{Gal}(K)$ there exist l such that $A_l(\tilde{K}(\sigma)) \neq 0$. This completes the proof of a conjecture of Geyer-Jarden from 1978 in characteristic 0.

A complete classification of cubic function fields over any finite field

Sophie Marques

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During this talk, we will present a complete classification of cubic function fields over any finite field, particularly developing a complete Galois theory which includes those cases when the constant field is missing certain roots of unity. In doing so, we will be able to describe easily ramification and splitting data from the generating equation, in analogy to the known theory for Artin-Schreier and Kummer extensions. In particular, we will also describe integral bases, and Galois actions in terms of canonical generating equations.

New results for compositions of numbers inspired by P. A. MacMahon

Augustine Munagi

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We give an account of new results for integer compositions, or ordered partitions, in the number-theoretic spirit of Percy A. MacMahon. The crux is the conjugation of a composition by graphical and algebraic methods. In recent years this approach has played a pivotal role in the discovery of composition analogues of classical partition identities while affording natural insights into difficult proofs. In this talk we develop the concept of high-order conjugation of compositions and generate new motivations for further research in the subject.

Parity considerations for certain partition functions

Darlison Nyirenda

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In this talk, we present some results on parity of certain partition functions and discuss related consequences. As part of consequences, a new recurrence for the number of partitions of a positive integer into distinct parts is derived. Various identities reminiscent of Legendre's partition-theoretic interpretation of Euler's Pentagonal Number Theorem are presented.

Simplicial homotopy theory of algebraic varieties over real closed fields

Ambrus Pal

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First I will introduce the homotopy type of the simplicial set of continuous definable simplexes of an algebraic variety defined over a real closed field, which I call the real homotopy type. Then I will talk about the analogue of the theorems of Artin-Mazur and Cox comparing the real homotopy type with the tale homotopy type, as well as an analogue of Sullivan's conjecture which together imply a homotopy version of Grothendieck's section conjecture. As an application I show that for example for rationally connected varieties over any real closed field the map from connected components of points to homotopy fixed points is a bijection.

Graph laplacians and Drinfeld modular curves

Mihran Papikian

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The relationship between combinatorial laplacians and automorphic forms is an active area of current research with applications to a variety of problems arising in number theory, group theory, and coding theory. We will discuss certain combinatorial laplacians arising in the theory of Drinfeld modular curves, and their applications to estimating congruences between automorphic forms.

Potential density on Calabi-Yau threefolds

Fabien Pazuki

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Let X be a projective variety defined over a number field k. We say that X has potential density if there exists a finite extension k'/k such that the rational points $X(k')$ are dense in X for the Zariski topology. We report on a joint work with Bogomolov, Halle and Tanimoto where we prove that Calabi-Yau threefolds over number fields admitting double fibrations in abelian surfaces satisfy the potential density property. We will also give explicit examples of such threefolds.

Analytic interpolation of Thakurs multiple zeta values

Federico Pellarin

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The aim of this talk is to discuss the basic properties of multiple zeta values in certain Banach algebras of positive characteristic that analytically interpolate Thakur's multiple zeta values. We will review their basic properties: sum-shuffle relations, algebra structure, linear relations etc. and we will present some open questions and conjectures.

Finding Endomorphisms of Drinfeld modules

Richard Pink

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The endomorphism ring of any given Drinfeld A-module over a field is a finitely generated A-module. In principle it can therefore be described explicitly by a finite list of generators. We discuss what is involved in finding these algorithmically.
See `https://people.math.ethz.ch/~pink/ftp/KuhnPink20160809.pdf`.

On r-generalized Fibonacci numbers and associated transcendental numbers

F. Patrick Rabarison

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This is part of joint work with Arthur Randrianrivony and Tojo Andrianoela. The r-generalized Fibonacci numbers are like the Fibonacci numbers, but instead of starting with two predetermined terms, the sequence starts with r predetermined terms and each term afterwards is the sum of the preceding r terms. In the first part, starting from properties of Fibonacci numbers, I will present some arithmetic and combinatoric properties of r-generalized Fibonacci numbers. In the second part, after investigating basic and general properties of automatic sequences, I will present some applications and particularly, the construction of transcendental numbers. Explicit examples of the construction will be given and I will present a Conjecture that may lead to an uncountable family of transcendental numbers.

On the bound of ramification of crossed homomorphisms.

Cynthia Ramiharimanana

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Let $(K/K_0; G; \alpha)$ be an embedding problem with Kernel a simple $\mathrm{Gal}(K_0)$-module $A = C_l^r$. Under certain conditions, the local-global principle provides a solution ψ_0 of $(K/K_0; G; \alpha)$. To obtain a bound on the ramification of the solution which depends on r, one adjusts ψ_0 by multiplying it by a crossed homomorphism $\chi : \mathrm{Gal}(K_0) \to A$ such that the ramification of χ itself is bounded. In this talk, I will present the steps of the construction of χ with a bound on its ramification by using the corestriction map.

Strong Approximation Theorem over the Compositum of all Symmetric Extensions of a Global Field

Aharon Razon

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Let K be a global field, $\mathcal{V}$ a proper subset of the set of all primes of K, $\mathcal{S}$ a finite subset of $\mathcal{V}$, and $\tilde{K}$ (resp. K_s) a fixed algebraic (resp. separable algebraic) closure of K. Let $\mathrm{Gal}(K) = \mathrm{Gal}(K_s/K)$ be the absolute Galois group of K. For each $\mathfrak{p} \in \mathcal{V}$ we choose a Henselian (respectively, a real or algebraic) closure $K_\mathfrak{p}$ of K at $\mathfrak{p}$ in $\tilde{K}$ if $\mathfrak{p}$ is non-archimedean (respectively, archimedean). Then, $K_{\mathrm{tot},\mathcal{S}} = \bigcap_{\mathfrak{p}\in\mathcal{S}} \bigcap_{\tau\in\mathrm{Gal}(K)} K_\mathfrak{p}^\tau$ is the maximal Galois extension of K in K_s in which each $\mathfrak{p} \in \mathcal{S}$ totally splits. For each $\mathfrak{p} \in \mathcal{V}$ we choose a $\mathfrak{p}$-adic absolute value $|\ |_\mathfrak{p}$ of $K_\mathfrak{p}$ and extend it in the unique possible way to $\tilde{K}$.

Let K_{symm} be the compositum of all symmetric extensions of K, i.e. the compositum of all finite Galois extensions with Galois group isomorphic to S_m for some positive integer m. Then, the field $K_{\mathrm{tot},\mathcal{S}} \cap K_{\mathrm{symm}}$ satisfies the following local-global principle:

Let V be an affine absolutely integral variety in $\mathbb{A}_K^n$. Suppose that for each $\mathfrak{p} \in \mathcal{S}$ there exists $\mathbf{z}_\mathfrak{p} \in V_{\mathrm{simp}}(K_\mathfrak{p})$ and for each $\mathfrak{p} \in \mathcal{V} - \mathcal{S}$ there exists $\mathbf{z}_\mathfrak{p} \in V(\tilde{K})$ such that in both cases $|\mathbf{z}_\mathfrak{p}|_\mathfrak{p} \leq 1$ if $\mathfrak{p}$ is non-archimedean and $|\mathbf{z}_\mathfrak{p}|_\mathfrak{p} < 1$ if $\mathfrak{p}$ is archimedean. Then, there exists $\mathbf{z} \in V(K_{\mathrm{tot},\mathcal{S}} \cap K_{\mathrm{symm}})$ such that for all $\mathfrak{p} \in \mathcal{V}$ and for all $\tau \in \mathrm{Gal}(K)$ we have: $|\mathbf{z}^\tau|_\mathfrak{p} \leq 1$ if $\mathfrak{p}$ is archimedean and $|\mathbf{z}^\tau|_\mathfrak{p} < 1$ if $\mathfrak{p}$ is non-archimedean.

For $\mathcal{S} = \emptyset$, we get as a corollary that the ring of integers of K_{symm} is a Bezout and Hilbertian domain.

Algebraic approaches to the Brauer-Siegel ratio for abelian varieties over function fields

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In analogy with the classical Brauer-Siegel theorem, Marc Hindry proposed the study of the ratio

log((order of sha)(Neron-Tate regulator)) / log(exponential differential height)

for families of abelian varieties over a fixed global field. Later, Hindry-Pacheco and Griffon proved a number of interesting results about this ratio, including a computation of the limit for several natural families, over global function fields of characteristic p. Their arguments use the formula of Birch and Swinnerton-Dyer for the leading term of the L-function of the abelian variety and analytic arguments. We will explain how to recover (and extend) some of their results by algebraic means, namely by a direct analysis of the Tate-Shafarevich group.