A Correction

by

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Recently, I discovered a rather ridiculous error in my paper, *Ueber transzendente P-adische Zahlen*, Comp. Math. 1935, 259—275. On page 268 of this paper, the following statement is made:

„*Let* α *be a* P*-adic integer different from* 0 *and* 1, *and such that*

$$(1) \qquad |\alpha - 1|_P \leqq P^{-1}.$$

Then $$A = \log \alpha$$

satisfies the inequalities

$$(2) \qquad 0 < |A|_P \leqq \begin{cases} P^{-2} \text{ if } P = 2, \\ P^{-1} \text{ if } P \geqq 3." \end{cases}$$

If the prime P is odd, the statement is quite correct; and the *upper* bounds in (2) remain true for $P = 2$. *But if* $P = 2$, *then the condition* (1) *does not imply that* $A \neq 0$. For evidently the diadic number

$$(3) \qquad \log(-1) = \log(1-2) = -\sum_{n=1}^{\infty} \frac{2^n}{n}$$

vanishes since

$$(-1)^2 = +1 \text{ and therefore } \log(-1) = \tfrac{1}{2}\log 1 = 0.$$

There are, however, no further zeros of the diadic logarithmic function.

For assume that

$$\alpha \neq 0, \quad \alpha \neq 1, \quad |\alpha - 1|_2 \leqq \tfrac{1}{2}.$$

Then $$\alpha \equiv \mp 1 \pmod 4,$$

and since $\log(-\alpha) = \log \alpha + \log(-1) = \log \alpha$,

we may assume, without loss of generality, that

$$\alpha \equiv +1 \pmod 4,$$

hence that $$0 < |\alpha - 1|_P \leqq \tfrac{1}{4}.$$

Therefore

$$\log \alpha = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(\alpha-1)^n}{n} \equiv \alpha - 1 \pmod{2(\alpha-1)}, \text{ i.e. } \log \alpha \neq 0,$$

as asserted.

Hence, if $P = 2$, the two numbers α and β of my paper must be different not only from 0 and 1, but also from —1; but with this small further restriction, the proof of transcendency becomes again valid.

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