Review of 'Loving and Hating Mathematics' to appear in the *Mathematical Intelligencer*

Jonathan M. Borwein and Judy-anne Osborn
Centre for Computer Assisted Research Mathematics and Its Applications
The University of Newcastle, Australia

September 20, 2011

1 Myths & Mathematics

Loving and Hating Mathematics (hereafter referred to as *Loving and Hating*) is the child of two passionate scholars: a mathematician and a social scientist. Reuben Hersh will be known to some readers for his many articles in the Intelligencer, as well as earlier books such as *The Mathematical Experience* coauthored with Davis and Marchisotto, and *What is Mathematics Really?* The latter had a substantial effect upon the older of the two reviewers, being, at the time of publication, a welcome blast of mathematical humanism.

The present book, *Loving and Hating*, is written in the same clear gentle style, and has as its expressed aim the vanquishing of four myths:

1. *Mathematicians are different from other people, lacking emotional complexity.*
2. *Mathematics is a solitary pursuit.*
3. *Mathematics is a young man's game.*
4. *Mathematics is an effective filter for higher education.*

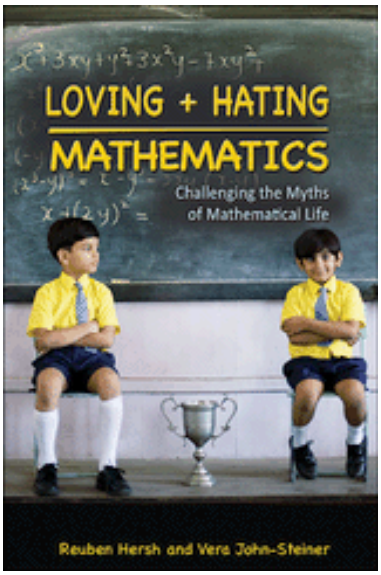
Outline of *Loving and Hating*

Loving and Hating has chapters addressing mathematical: beginnings, culture, solace, addictive potential, communities, gender and age related issues, philosophies of teaching of mathematics in Universities, and, last of the numbered chapters, teaching of mathematics in schools. At 416 pages, it is as compact as it could be, given the ambitious breadth of its scope.

Only the last and first chapters of the book deal directly with school mathematics. This makes the cover design due to Lorraine Betz Doneker, which is clearly school-situated, particularly worthy of mention. The image is emblematic of a key message of the book, which the authors express as follows as they conclude their last chapter: *Because we love mathematics, we want to minimize the num-*

ber of those who hate it. The picture is of two adorable little boys in short-trousered school uniforms (bringing back the first author's memories of his own school uniform). The two sit in front of a blackboard, with a silver trophy between them. One is forward facing and looks quite content, the other notably unhappy and gazes to the side, at the first. The children pictured personify the *Loving* and *Hating* of the title, as was underlined to the authors of this review when a young person who glanced at the book instantly associated himself with the sad-looking protagonist.

The book is not structured as a recipe to address problems with school experiences of mathematics, however. Rather it is a tour of mathematical life in the large, carrying with it a recommendation that design issues relating to the school-level experience of mathematics should be addressed in terms of mathematics in its entirety, and in particular the joy that its practioners take in the endeavour.



The chapter titles and starting page numbers are as follows:

Chapter 1: Mathematical Beginnings 9

This title could mean many things - as it happens the authors address how a child becomes engaged in mathematics. To an extent this picks up on the trophy on the front cover, since it includes a section on mathematics competitions. We learn of the childhood mathematical experiences of famous mathematicians such as Terence Tao, Carl Friedrich Gauss, Sonia Kovalevskaya and many others; as well as observations of personality and psychological issues recurrent in childhood enjoyment of mathematics.

Chapter 2: Mathematical Culture 46

This chapter makes clear that mathematics is a subject with a culture reaching back over a long long time. The authors' description encompasses thoughtful forays into four main ideas: abstraction, aesthetics, belongingness and the tension between collaboration and competition.

Chapter 3: Mathematics as Solace 89

The authors ask: *"Is mathematics a safe hiding place from the miseries of the world?"* in this chapter, and answer that it can be. They illustrate that the meaning of the claim ranges from an absorption which temporarily keeps the worries of the world at arm's length, to a means of coping with situations as extreme as imprisonment.

Chapter 4: Mathematics as an Addiction: Following Logic to the End 106

Mathematical researchers tend to have a sense of what 'mathematics as addiction' means, for better or for worse. Here, we get a sample of some of the extremes: after a mention of John Nash, whose life and schizophrenia was the subject of the book and movie *A Beautiful Mind*, a detailed picture is painted of the extraordinarily creative and intense life of Alexander Grothendieck. Following this, the authors present 'five cases of actual criminal or suicidal insanity in other mathematicians', writing about famous cases including the tragic later life and death of the renowned logician Kurt Gödel.

Chapter 5: Friendships and Partnerships 138

This chapter describes some famous friendships between mathematicians: Karl Weierstrass and Sonia Kovalevskaya, the trio of Hardy, Littlewood and Ramanujan, and the friendship between logician Kurt Gödel and physicist Albert Einstein; amongst others. Mathematical marriages such as between Julia Bowman Robinson and Raphael Robinson are also described - this relationship is also described in the book *Julia* and film *Julia Robinson and Hilbert's Tenth Problem*. The importance of friendships and partnerships in sustaining the individuals involved is described both in particular and in general.

Chapter 6: Mathematical Communities 176

Communities of mathematicians that have formed spontaneously or in organized ways, to meet the needs of the groups that comprise them, are described. Examples range from over a century old to the present, including the faculty at the University at Göttingen in Germany (1890's-1930's), the famous French group Bourbaki which began in the 1930's, the short lived Jewish People's University (1978-1983) in Moscow, and contemporary examples such as the Association for Women in Mathematics (AWM) and the web-supported Polymath Project. The ways in which the communities support their members, in which the communities themselves die or flourish, and in which these groups together are part of a larger mathematical community, are described.

Chapter 7: Gender and Age in Mathematics 228

This chapter addresses mathematical life through the lens' of gender and of aging. The experiences of many famous women mathematicians are described, including historical examples in which being female was a considerable impediment to mathematical life, such as experienced by Sonia Kovalevskaya and Emmy Noether. Notably absent amongst the historical examples is Lady Ada Lovelace who is famous for her work on algorithms and information, in connection with Babbage's analytical machine.

The varied experiences of contemporary

women mathematicians such as Karen Uhlenbeck, Joan Birman and Fan Chung are described. Again, this is a selection — pleasingly there are now too many accomplished women in the profession to be comprehensive — and does not include other equally notable women such as Cathleen Morawetz, nor any non-Americans. The experience of being a mathematician and getting older is also described in its variousness in some detail, reprising the results from an earlier published survey conducted by Hersh as well as comments from other surveys.

Chapter 8: The Teaching of Mathematics: Fierce or Friendly? 273

The focus in this chapter is upon just two University level examples: the systems exemplified by Robert Lee Moore (Moore Method) and Clarence Francis Stephens (Potsdam Model). These examples are extremes points, both from the USA, rather than the sort of barycentric averages that may be common practices now inside and outside of the USA. As the authors of *Loving and Hating* write, the two models 'embody two different, opposed strains in American Education: the egalitarian versus the elitist; the cooperative versus the competitive; the heritage of the Declaration of Independence versus the heritage of the Confederate States of America.'

Chapter 9: Loving and Hating School Mathematics 301

The final chapter begins with observations about the effects of school education in mathematics upon the feelings that adults have towards mathematics, including the observation that these feelings often include the *'hating'* of the book's title. The chapter includes relatively little description of mathematics in the classroom as experienced by school students. There is reference to mathematical learning in a variety of contexts, such as the shopping contexts investigated by anthopologist Jean Lave. The chapter includes many suggestions for reform in the teaching of mathematics, with reference to trial programs in various contexts. A thread underlying many of these suggestions is the idea that people have multiple different kinds of intelligences, and that teaching generally should not privilege mathematical thinking or even specific kinds of mathematical thinking.

End matter

Following the numbered chapters are five pages of conclusions, nine pages of 'literature review' listing other popular books on mathematics, and thirty four pages of paragraph long biographies of mathematicians. The last mentioned compendium was of particular interest to the (mathematician) husband of one of us, who picked up the book upon its arrival in the household, and when he discovered the biographies at the end, sequestered it until he had read them and the rest of the book through. This biographical 'digestif' to the book with its overview of the lives of both well known and less known mathematicians may be one of the highlights for those readers who are themselves part of the mathematical community.

To summarize, *Loving and Hating* is a sweeping survey of mathematical life, into which the four myths and the antidotes the authors provide are woven. We structure the remainder of our review around the following two sets of questions, which arose for us in the reading.

About the *myths*

1a. Are the four claims actually *myths*?

1b. Who believes them?

1c. Are these *myths* about mathematicians, or about broader groups?

About the audience

2a. To whom is the book addressed by the authors?

2b. To whom would the book be useful?

2c. Will the book *Loving and Hating* find its audience?

About us

We are both research mathematicians, with interest and experience in communicating mathematics with the general public as well as students and peers, one of us late-career and the other early-career. Laureate Professor Borwein has been involved in mathematics outreach in four countries on three continents over a period of nearly forty years. Dr Osborn has led community-building and outreach activities at the Australian National University and the University of Newcastle. Both of us are currently employed at the University of Newcastle in Australia, in the centre for Computer Assisted Research Mathematics and its Applications (CARMA), of which Professor Borwein is the director.

As reviewers of *Loving and Hating*, we find ourselves largely in agreement; this review being a snapshot of our discussions. For much of the review we write in explicit dialogue (**JB** for Jon Borwein and **JO** for Judy-anne Osborn), to clarify our different perspectives and occasional disagreements. Where we write in one voice, we agree with each other.

2 The *Myths* one by one

Myth 1

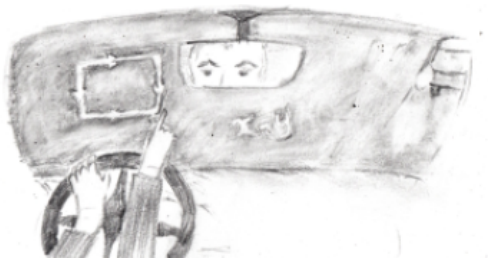
Mathematicians are different from other people, lacking emotional complexity

Is this a widely held belief? Does it have a basis in fact? Hersh and John-Steiner have

run two claims together here, and we wonder if the result is obfuscation?

JB: If *Myth 1* is meant to say that mathematicians are 'a bit odd', then its widely believed, and often true. Otherwise it might not be true. In my father's house I grew up around many mathematicians, who ranged from the urbane and articulate to the seemingly mute.

JO: Yes - indeed quite a few anecdotes in *Loving and Hating* reinforce rather than diminish a perspective of eccentricity. For instance in Chapter 2: Mathematical Culture, R H Bing is described as driving colleagues to a conference, and when the windscreen fogged up, used it



to draw mathematical diagrams on, rather than wiping it clean.

JB: Films like *A Beautiful Mind* pick up on and emphasize the idea of the eccentric or insane mathematician. It *is* a myth that being crazy helps a person to do good mathematics (or much anything else): it doesn't. As Michael Crichton had said, however, "All professions look bad in the movies - why should scientists expect to be treated differently?"

JO: Evidence countering the second part of the claim - that mathematicians lack emotional complexity - is overwhelming, as reflected by Hersh and John-Steiner in story after story. Emotions such as attachment, affection, joy, courage, fear, empathy, anxiety, sorrow, indignation, depression and wonder, are related in many accounts of discoveries, friendships, prison terms, politics, competition, collaboration and every-day life. I was struck with a sense of recognition when I read in Chapter 2 of the joy that Professor Jenny Harrison finds in nature, exploring paths through the woods; and looking at mathematical landscapes with something of the same feeling.

JB: Yes, the sense of *wonder* is palpable in Grothendieck's description of his feelings when he switched mathematical fields from analysis to geometry:

> *It was as if I had fled the harsh arid steppes to find myself suddenly transported to a kind of 'promised land' of superabundant richness, multiplying out to infinity wherever I placed my hand on it, either to search or to gather ...*

I was also taken by the description of Chandler Davis' response to his six months in prison, which resulted from his refusal to cooperate in some McCarthy era questionings by the Committee on Anti-American Activities. Davis'

sense of humor, and courage, is expressed in a footnote to one of his subsequent papers:

> *Research supported in part by the Federal Prison System. Opinions expressed in this paper are not necessarily those of the Bureau of Prisons.*

We are told that the delicate wording of this gem was suggested to Davis by a friend. The story is an instance of how many things can happen to a principled person in a long life.

JO: In terms of who the audience for this book is, this *myth* under discussion makes me think that it must be aimed at the general public, for surely the belief that 'mathematicians lack emotional complexity' could only be held by members of the public who don't happen to know any mathematicians?

JB: I am not so sure. The part on Grothendieck's work, for instance, is far too technical for the lay-reader.

JO: I agree that the detail on Grothendieck's work is very technical. For myself as a mathematician I found it required more concentration than I was willing to give at the time. Yet surely it gives a flavor of a kind of mathematics, in a way that would be impossible otherwise, and lets people who have never trodden the halls of a University Mathematics department have a sense of what it is like, even without understanding details. I think that Hersh and John-Steiner have aimed to write a book which has something for both lay-readers and mathematicians alike.

JB: Much of what is being put forward in *Loving and Hating* as being unique to mathematicians, applies to any group of people who pursue a life of the mind. As for characteristics such as madness or emotional range, I see no difference with Physicists or with English scholars, for instance. (It may be that there is an autistic tendency in mathematicians - quantitative studies could measure this, and maybe even distinguish mathematicians from physicists.) However in the main, just as mathematicians are portrayed as mad in the movies, the idea of the 'mad poet' is a romantic concept. There is no reason to think that fewer Physicists or Writers go off the deep end.

JO: Famous physicist Ludwig Boltzmann and famous author Virginia Wolfe?

JB: Yes, those instances and more.

JO: Surely there's no harm in just focusing on mathematicians, in a book about mathematical life?

JB: *Loving and Hating* would be better for more situating. It is misleading, for instance, to write about the Unabomber as a mathematician, in the chapter on Mathematics as Addiction, without talking about other scientists who also did

crazy violent things. I searched the Unibomber's massive manifesto in Altavista at the time it was published. I found the word mathematics only occurred in the phrase "science and mathematics" and that only three times.

JO: I asked my sister, as a person outside of academia, whether she thinks that *Mathematicians are different from other people, lacking emotional complexity*? My sister replied that she does not think that most people think mathematicians lack *emotional complexity*. She said that she thinks that the general public think of mathematicians as people who spend time writing strange complicated things on bits of paper.

JB: That's an interesting reply.

JO: Yes, so was *Myth 1* ever a myth at all?

JB: It has the weakest claim of the four to be a myth. What is true is that many mathematicians have interesting stories. However, any reader of *Loving and Hating* who begins the book with the view that mathematicians are uniquely and exclusively passionate about their particular field, and idiot savant in everything else, will come away knowing that is not the case.

Myth 2

Mathematics is a solitary pursuit.

JO: It is not very surprising that people outside of mathematical research would think of it as a solitary pursuit, since it is often depicted in the movies as being the work of a lone genius in their own private ivory tower replete with chalkboard.

JB: The movies aren't the only source for such a view. It is a default perception of creative work, if there isn't reason to think otherwise - we also think of novelists as working in solitude.

JO: Yet in mathematics, the idea of solitary creation is only a half-truth. Excellent mathematical work can be done in solitude (for instance as the half dozen or so examples of mathematicians in prison in the book testify), but this condition is not the norm. Interaction between

mathematicians, both planned and accidental, with sharing of ideas and mutual inspiration, is typically important or necessary in mathematical creativity.

JB: I think that we're agreed that *Myth 2* is a myth: a belief both widely held, at least outside of mathematical circles, and in the large not true. That said, it is still true that we have no technology for telepathy, so that the creative thinking that is essential to mathematical progress is still necessarily an individual part of the activity.

JO: Yes, indeed, and Hersh and John-Steiner do a good job of rebutting the myth in the large. In Chapter 5 on Friends and Partnerships, I was fascinated to read of the friendship between Hilbert, Minkowski and Hurwitz, and how the latter two helped Hilbert plan his famous '23 important open problems' talk which he gave in Paris in 1900, which has shaped much of mathematics for the following hundred years.

JB: A story I find resonant is that of Massera, since my colleague at Carnegie Melon, Massera's student Shaeffer, was heavily involved in the campaign to free Massera when he was interned in prison in Uruguay for 9 1/2 years. Massera was a collaborative and generous person both in and out of prison. As a prisoner he was involved in circulating forbidden papers on dialectic, logic and mathematics that helped the prisoners keep their spirits up. It is a fine example of humans supporting ideals and each other in hard times.

JO: Another interesting instance of a cooperative spirit, in an entirely different context, is Timothy Gower's 2009 PolyMath project, described in Chapter 6, in which a major open mathematics problem was posed on the web, a collaborative effort inspired, and the problem solved in the astonishingly short time frame of a few weeks by "the shared effort of over two dozen contributors from several countries."

JB: The internet has made an astonishing difference to the practicality of multi-location collaboration. In Chapter 1 Hersh and John-Steiner comment that *students receive guidance and inspiration not only face to face but also at a distance*. I found it curious that they chose to illustrate this with a century-old example of Birkhoff in the USA being inspired and learning from Poincaré in Paris. Yes, collaboration and mentorship has been important in mathematics throughout the ages, but there are many more contemporary examples — such as the role of the arXiv — that they could have drawn upon.

For instance, when I was still at Simon Fraser University in Vancouver in 2003, the one of the biggest power blackouts for a generation in Northern America, occurred. In our research centre we knew instantly that there was a problem, when none of our contacts east of the Missouri

would come up. In fact, we realized that we were more quickly aware of a problem affecting our distant colleagues in our discipline, than we would have been if there had been major trouble in part of our own physical campus.

JO: The support that mathematicians can give each other is beautifully described in a quote about Chern in his capacity as a thesis supervisor, who

> *... conveyed the philosophy that making mistakes was normal and that passing from mistake to mistake to truth was the doing of mathematics. And somehow he also conveyed the understanding that once one began doing mathematics it would naturally flow on and on.*

JB: About tenn years after my thesis I started collaborating seriously with other scientists, and discovered that is not just from other mathematicians that inspiration can be found. I have come to the realization that how good you are at formalizing 20th century rigorous expressions is not a complete measure of your mathematical worth. I would often rather collaborate with a Physicist on a poorly posed problem, than with a mathematician close to my own field, because there may be more undiscovered nuggets in the bringing together of two more different perspectives.

JO: That's something that doesn't shine through so much in *Loving and Hating* - the rich mathematical opportunities for mathematicians in collaboration with other scientists.

JB: Yes, I'd like to see a book which does for the sciences what *Loving and Hating* attempts to do for mathematics. What would such a book look like? The closest that I can think of at the moment is *A Passion for Science*, by Alison Richards and Lewis Wolpert. I am coming to think that there may be more differences within mathematics, than there are across the sciences in their entirety.

Wolpert interviewed Christopher Zeeman who told a story about one of his (non-mathematician)) administrators who helped run an annual summer conference series in Warwick, which rotated every three years between the three mathematical areas of Algebra, Topology/Geometry, and Analysis. She said that after a while she could tell which was the year's theme without looking at the program, but just by observing the behaviour of the participants. The Algebraists were punctual, organized and thrifty. They wanted single cheap rooms and arrived by train when they said that they would. The Topologists wanted big houses, they brought their families and wanted to stay the whole week. The Analysts were predictably unpredictable, promising to turn up Tuesday with their partner and arriving on a different day from somebody else.

Maybe the main cleavages are within our discipline? There are differences in levels of rigour. Perhaps there are differences in levels of embodiment? The book, in its refutation of the myth of solitude and more generally, would be better for more contextualizing.

JO: I am reminded of the characteristics that Hersh and John-Steiner describe in Chapter 1 as reflective of the whole tribe of mathematicians: curiousity, determination, willingness to spend time doing mathematics, not minding being alone so much as others might, cherishing independence, and having a love for symmetry or logic or pictures or, sometimes, how things work. It strikes me that the characteristic of willingness to be alone is the grain of truth in the myth, that gives it some of its traction.

If Hersh and John-Steiner had included all the contextual comparison with the rest of science that you want them to, it would likely have been a much bigger book, whereas currently it is just a comfortable size for carrying and reading on the train.

JB: The characteristic of being willing to spend time alone thinking isn't special to mathematicians or even scientists more generally. It is a property of leading a *life of the mind*, and that is a point worth making in a book which endeavors, at least in part, to introduce the wide public to what a mathematical life is like.

Myth 3

Mathematics is a young man's game.

JO: Hardy wrote that 'Mathematics is a young man's game' when he was in his sixties and in a self-confessed melancholy mood, writing *about* mathematics instead of *doing* mathematics, which he would have preferred.

Hersh and John-Steiner address this *myth* in their Chapter 7: *Gender and Age in Mathematics*, where they make it rightly clear that Hardy would have meant 'person' by 'man', and that he was making a claim about age, not women. Their chapter deals with issues of both gender and age; here I focus on 'age'.

Is it widely believed that mathematics is 'a young person's game?' And is it true?

JB: This is widely believed, both within and outside of the mathematical community.

JO: I agree with you that this view is prevalent inside the mathematical community. I disagree about outside of it. For many of the general public, the only person that they envisage as a mathematician is Einstein, and their picture is of a brilliant old man with a shock of white hair. I think that the public myth of "genius" is of "Age and Wisdom".

JB: If that's the case, then the public have forgotten what a media star Einstein was when he burst onto the scene in his youth.

JO: Is it true that mathematics is 'a young person's game'?

JB: There's an aspect of truth to it in many fields, not just mathematics. There are young geniuses and old masters, as is related in the piece on artistic genius in my book with David Bailey about Experimental Mathematics. The difference is like that between an early Picasso cubist work and a Renaissance masterpiece. Breakthroughs tend to be made by the young.

JO: It is related in *Loving and Hating* that Hardy's own long-time collaborator Littlewood was to become a counter-example to Hardy's claim, publishing a "monster" paper with Mary Cartwright that "was recognized as an early breakthrough in the discovery of chaos", when Littlewood was in his seventies.

JB: Yes, if you maintain a passion for your field, as I do, and reasonable health then you can continue to make fine contributions as you age. I can still do good mathematics in my sixties. My father enjoys doing mathematics at much at 87 as he did at 18.

The myth with the strongest credos within the mathematics community is really:

> *Doing first rate research mathematics is something that you had better hurry up and do before you're 35 or 40.*

It is very unusual to find someone who has been a toiler in the 'mathematical vineyards' who suddenly has a huge result at age 50.

JO: The mathematician Mary Ellen Ruden is quoted in the book as stating in interview

> *I don't think most people's best work will be done by the time they're thirty, and certainly my best work wasn't done until I was fifty-five years old.*

Might the myth you state be a self-perpetuating myth? For instance only mathematicians not over 40 years of age are eligible for the Fields Medal, which is the mathematical equivalent of a Nobel Prize.

JB: In fact the Fields medal is awarded for the contribution that the person has made *and is expected to continue* to make!

JO: Oh that's interesting, and it fits with the responses to Hersh's survey of his fellow mathematicians. Many say that they continue to make good progress, in their later years, though their mix of skills and strategies tends to change.

JB: Yes, some mathematicians make more and more impact as they get older, whilst others burn out. Again, look at other creative fields - the synthetic rather than the accretive ones. Look at 'geriatric rock'. The *Rolling Stones* are still energetic, creative musicians.

JO: We've discussed how the book and our life experience, especially yours, topples the myth that excellent mathematics is only done by young people. But the book also tackles the claim that if you're any good at all as a mathematician that its going to show up before you turn 40, and intriguingly, much of the data refers to women.

For instance, I quote from Chapter 7:

> *Joan Birman, a topologist at Columbia University-Barnard College, did not get her PhD until she was 40 years old. Birman focused better on math after the issues of marriage were sorted, her children older, etc. "I think doing mathematics when you're enthusiastic is important – not your age."*

If it is true that women often make their first good achievements later on, it is consistent with what my (male) thesis advisor used to tell me. He didn't believe that mathematicians become notably worse as they age, but rather that there are more other responsibilities and distractions that tend to get in the way of creative achievement. As I remember it, he told me that if you can keep yourself clear of the 'crap', you can still create.

JB: Yes and yes. I learned to keep the'crap' under control from my father who was a research active mathematics Department head for a very long time. My mother who got an anatomy PhD the year before I got mine was a wonderful example of deferred female achievement.

Myth 4

Mathematics is an effective filter for higher education.

JO: It is said that the door to Plato's Academy was engraved with the phrase:

Let none ignorant of geometry enter here.

Whether or not this fable is true, the spirit of the idea is consistent with Plato's conception of mathematics as a means to train the mind, an idea which has been embedded in western thought ever since.

JB: Yes, historically in Cambridge the earliest, and for along time only, examined degree awarded was in Mathematics. It was called the *Tripos*. Keynes did the Tripos, as did Airy, Herschel, and many other famous people in and out of the profession of mathematics.

JO: Such as the economist Thomas Malthus; astrophysicist Arthur Eddington; discoverer of argon Lord Rayleigh; founder of the theory of electromagnetism James Clerk Maxwell; philosopher, logician, mathematician, historian, and social critic Bertrand Russell?

JB: Indeed - there is an illustrious history. At much the same time the degree of equivalent standing at Oxford University was called *Greats* (or *Classics*), an archetypal humanities degree, emphasizing literature, language, philosophy, history and art, and was the course taken by aspiring ministers of the church, social thinkers, writers, politicians and civil servants.

JO: So the special status of the subjects *Mathematics* and *English* in schools through-out the English speaking world, is inherited from the two great English Universities of the Middle Ages?

JB: In many ways, yes. Even now, although individuals claim to 'hate math', or not be able to do it, nobody doubts its importance.

JO: I am not so sure.

JB: No serious business at any serious level is functioning these days without an enormous amount of mathematics. The trend is reported on in publications such as *Business Week*, with cover stories with titles like, "Top Mathematicians are becoming a new global elite" in January 2006.

Successful tech companies, such as *Nortel* (originally *Northern Bell*) was a decade ago, have been successful partly because of employing mathematically trained people. At the height of its success, the only factor that they could identify to distinguish successful from less successful research groups was that those with mathematicians in them tend to be more productive.

JO: That understanding may not be as broad as you think, particularly in schools. Students are asking, *What is the point?* and, *When am I ever going to use this?*

JB: Yes, in Chapter 9 of the book, the mathematician Underwood Dudley is quoted as arguing that algebra and calculus are seldom used or needed by most people.

JO: Hardy made the same point with more eloquence in his famous 'Apology':

> *... some mathematics is certainly useful in this way; the engineers could not do their job without a fair working knowledge of mathematics, and mathematics is beginning to find applications even in physiology. ... It is useful to have an adequate supply of physiologists and engineers; but physiology and engineering are not useful studies for ordinary men.*

JB: Since Hardy's day (he was born in 1877 and died shortly after the end of the second world war), the professions that rely on mathematical thinking have multiplied. I think he would have been greatly amused at the number of number theorists who work for the security agencies around the world such as CSIS or NSA. That said, I do not believe that we teach the right mathematics to the right students. The problem is that we are bad at identifying the right students.

As an eleven year old in Britain I experienced something called the '11 plus'. It was an examination taken by all British children, which determined at that young age the rest of my academic future by specifying what kind of secondary school I was admitted to. The pressure on my eleven-year old self was horrendous, even though I was capable in doing well in that exam and did so. I do not advocate the '11 plus' or anything like it, yet in its absence, I challenge anyone to find a diagnostic for those who will need mathematics beyond arithmetic better than letting everyone try it.

JO: You're saying that we need engineers and so forth, and that they need mathematics, and - I'm inferring here, since mathematics is cumulative and takes a long time to learn - we had better ensure everyone does mathematics up to a certain level.

JB: Yes. There is an analogy with learning a foreign language. One of the few things that linguists agree about, is that learning a language before age eight is different to learning it afterwards. In the same way, it is very hard to lay down high-end mathematical skills after school; to say, 'I'm going to catch up at University'. I do know some counter-examples: one of my teachers Michael Dummett, the renowned British philosopher, taught himself mathematics so that he could understand Frege - but such counter-examples are rare.

JO: What about the claim that mathematics should be used as a filter to entry to professions that don't need it, as described in *Loving and Hating*?

JB: That is very different. However, a belief in mathematics as a general filter has

had considerable staying power in Universities. For instance, by and large, a student may not fail much else in a Business Degree, but he or she will need to get through Calculus 100 and something Actuarial.

JO: I am puzzled by what is meant by the phrase, *an effective filter*? Does it mean that a moderate proportion of students tend to fail first year Calculus, so that this can be used as a sieve independently of whether it is sieving on some sensible criteria?

Or does *effective filter* mean that an ability to do mathematics courses is being used as a proxy for general intelligence? People sometimes say to me, "Oh you do maths; you must be so smart". It strikes me as an odd comment, coming as it often does from people who do work which seems to me much more subtle and difficult, like the clinical practice of medicine. Perhaps the belief that mathematical capacity implies intelligence is widely held? I don't know about the other way around.

The authors of *Loving and Hating* counter the myth by quoting Gardner's theory of multiple intelligences, describing mathematical abilities as being distinct from and not necessarily related to other equally important intelligences such as linguistic, musical and interpersonal.

JB: We filter inappropriately. Many of the people who come to University would be better off in a more technical training, but as long as there is a (completely unjustified) status gulf between university and technical college, it won't happen. This is managed better in Germany I think.

JO: What is University for?

JB: Respected social scientists like Christopher Jenks would say that, in its degree-awarding capacity, a University is not so much about what students learn as it is about what employers learn about the graduand: that you're reasonably well socialized, that you can follow other people's rules. This is in contrast with the liberal arts idea of education. Degrees like *Classics* (*Greats*) at Oxford and the *Tripos* in Mathematics at Cambridge used to be a measure of general educational attainment, and people were hired on that basis for careers that had nothing to do with the subject matter. Even in the seventies you could see individuals advertising for a job in the London Times with a qualification of 'BA Oxon (failed).'

I had an old friend who tried to lose his history thesis draft on the London to Bristol train the day after he was hired by the British Home Office. He wanted a defensible excuse not to finish it! Likewise, most people coming to Universities to study these days are not coming with a passion for some given subject. There is however, an important minority who do. This is mixing chalk and cheese.

JO: I am deeply imbued with the liberal arts tradition, whose values are expressed beautifully in Alfred North Whitehead's 1930 essay on the 'Aims of Education', in which he wrote *The university imparts information, but it imparts it imaginatively. ... A university which fails in this respect has no reason for existence.* Based on my experience of learning and of teaching, I expect that students may be transformed and delighted by what they learn at University, as well as getting a qualification that may lead to a job, even if their original aim was the more tangible one only.

JB: We conflate the idea of the ability to do mathematics well with the ability to appreciate it. We don't make the same mistake with Shakespeare, or sports.

What I would like to be able to do, when teaching mathematics, is to ask students in the class who is there for appreciation, and who needs mastery for subsequent professional use. Those in the first group could take the course as Pass/Fail, and those in the second for a numerical grade. I don't mind which group individuals are in. Both are worthwhile. With modern technology there is more and more capacity to meet both kinds of needs in the same classroom.

JO: That's an arrestingly interesting idea. It is also, in hindsight, compatible with the suggestion by Hersh and John-Steiner that

> *There is a wrenching strain between opposing pressures: a continuing demand for enough sophisticated math specialists, with a shrinking need for traditional math skills in the general population. ... Studying [math(s)] should continue to be required, but not in such a manner that students remember it with antagonism and loathing.*

JB: Many of the points made in the last chapter are about what could or should be, as opposed to what is. This is a shift in style from the earlier chapters, which were more driven by personal experience and anecdote.

JO: Yes, there is an enormous idealism in the final chapter, which collects together lots of stories of ideas that individuals have for trying to begin to make a difference to the experience the school-children have of mathematics classes. The suggestions are less developed in this final chapter on school-level teaching, than are the two striking cases studies of the egalitarian methods of Stephen and elitist methods of Moore, presented in the penultimate chapter on University-level teaching.

Whether in the school or the University setting, personally I find myself drawn to both Moore's method (minus the racism and anti-feminism, of course!) in which he believed immensely in individ-

ual students' capacity to discover things for themselves; and Stephen's Potsdam Model based on his fundamental belief that

> *[A]ny college student who wanted to learn college mathematics could do so if the learning environment was favorable.*

Based on my experience of learning and of teaching, I find resonance in Stephen's oft-repeated phrase, *go fast slowly*.

Conclusions

Loving and Hating is a book full of gems. We found that we could open it on any page and find something interesting. It is imbued with the authors' love of mathematics and respect for people. The message that mathematics is a fundamentally human activity, in which people can find meaning and joy, is clearly conveyed.

The book has flaws. We liked the parts each in turn more than the whole. Whilst all mathematicians of a generally philosophical nature are likely to enjoy browsing in *Loving and Hating*, we are less sure if the order in which the material is presented is of service to the rest of its potential readership, including teachers, policy-makers, and general public.

We find ourselves much in agreement with Kevin McConway, whose review in Plus Magazine: `http://plus.maths.org/content/loving-hating-mathematics` concludes that,

> *This is a complex book, which does not entirely achieve what it set out to do, and which does not entirely hang together as an organised whole. Sometimes the richness of the anecdotes and case studies gets in the way of the overall messages. The stories are not going to convince someone to continue their mathematical studies if they are worried about becoming different from their non-mathematical friends. But it is worth reading for the admirable way that the stories and anecdotes humanise mathematics - and because many of the anecdotes are very good stories in their own right.*

We conjecture that amongst potential readers who are not professional mathematicians but who are interested in mathematics, the chapter on school mathematics may be placed rather late on the scene, given that for this group of potential readers one of the main touchstones of mathematical experience may have been in school. That said, the context in which the chapter on school mathematics is saved for last is that of making suggestions for improvements to the school experience of mathematics, where those suggestions are made on the basis of connecting 'school math' with the discipline at large described so well in earlier chapters. The immersion in mathematical life with its joys of discovery, which the earlier chapters provide, gives a fresh mindset for thinking about improving the experiences in the school classroom.

We do feel that this book may be of special interest to graduate students in mathematics, as part of an introduction to the stories and culture of the community that they are joining. The first author has a practice of recommending that his new graduate students read Lakatos' *Proofs and Refutations*, Medawar's *Advice to a Young Scientist*, Davis and Hersh's *The Mathematical Experience* and Yandell's *The Honors Class*; and is tempted to add *Loving and Hating Mathematics* to that list.

JB: Will the book find its audience?

JO: It is readable, informative, interesting: I think so.

JB: A school girl asked to review a book on penguins wrote:

> *This book told me a great deal more about penguins than I really wanted to know.*

If mathematicians are penguins and the readers are all marine ornithologists, this is great. Sadly, I am unconvinced that a broader audience will take the time to read this book. As a professional penguin, I think that this is a pity.

JO: Your point is so beautifully made that it is an exquisite pain for me to differ, but I do. I don't think that you have to be a bird-specialist to love penguins, or, for instance, to enjoy a well-made David Attenborough special on them.

Maybe Hersh and John-Steiner are not quite David Attenborough, but they're close. The descriptions in *Loving and Hating* are sympathetic and understandable.

The lives that Hersh and John-Steiner have led have allowed them to get up-close and personal with a species (mathematicians, and more generally people whose work is creative thinking) whose world many people don't ordinarily get to see, and may welcome a window into.

Acknowledgements

The second author thanks her sister and husband for helpful discussions and her former PhD supervisor for allowing his advice to be quoted. All the drawings in this article were produced by the second author.