A.B. Lucas Secondary School

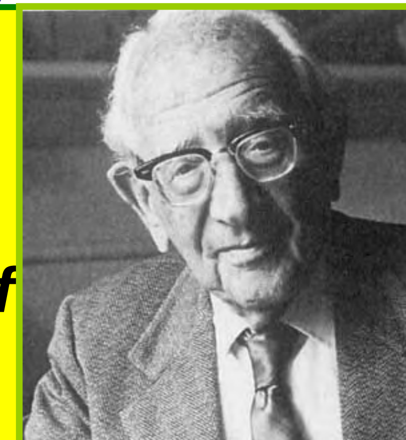
Dalhousie Distributed Research Institute and Virtual Environment

Math: What's New, What's Possible, What's Coming

Jonathan Borwein, FRSC www.cs.dal.ca/~jborwein

Canada Research Chair in Collaborative Technology

"intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication."



George Polya 1887-1985

AB Lucas Secondary School

December 12th 2007

Dalhousie University — Inspiring Minds

Faculty of Computer Science

Revised 12/12/2007

My Lab in Halifax

240 cpu Glooscap at Dal

Dalhousie Distributed Research Institute and Virtual Environment

D-Drive's Nova Scotia location lends us unusual freedom when interacting globally. Many cities around the world are close enough in a chronological sense to comfortably accommodate real-time collaboration.

Jon Borwein's Math Resource Portal

The following is a list of useful math tools.

Utilities

1. ISC2.0: The Inverse Symbolic Calculator
2. EZ Face : An interface for evaluation of Euler sums and Multiple Zeta Values
3. 3D Function Grapher
4. GraPHedron: Automated and computer assisted conjectures in graph theory
5. Julia and Mandelbrot Set Explorer
6. Embree-Trefethen-Wright pseudospectra and eigenproblem

Reference

7. The On-Line Encyclopedia of Integer Sequences
8. Finch's Mathematical Constants
9. The Digital Library of Mathematical Functions
10. The Prime Pages

Content

11. Experimental Mathematics Website
12. Wolfram Mathworld
13. Planet Math
14. Numbers, Constants, and Computation
15. Wikipedia: Mathematics

ICCOPT 2007 Short Course

16. Jon's Lectures

1. Moore's Law and Implications

"The complexity for minimum component costs has increased at a rate of roughly a factor of two per year ...

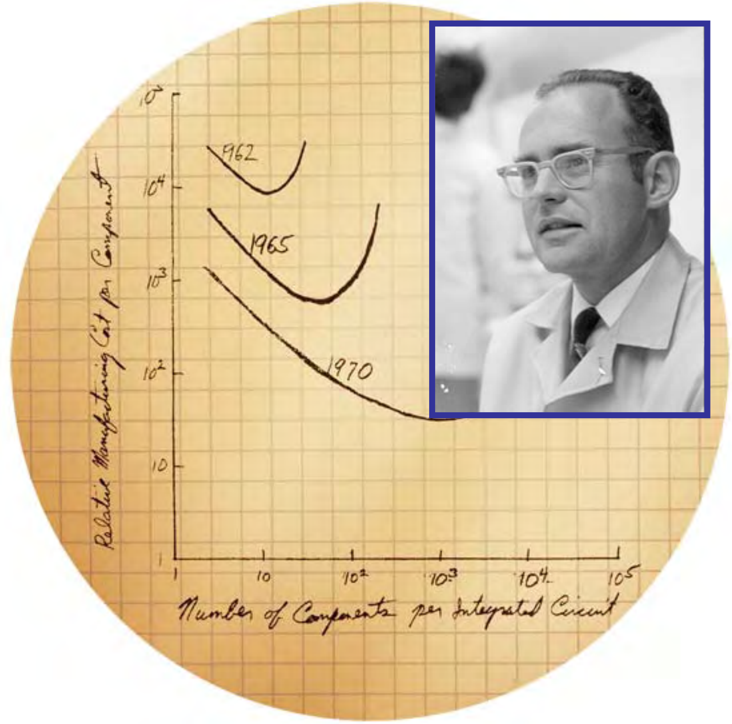
- now taken as "every 18 months to 2 years"

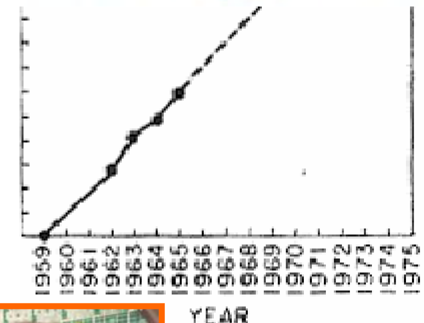
Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years. That means by 1975, the number of components per integrated circuit for minimum cost will be 65,000. I believe that such a large circuit can be built on a single wafer.

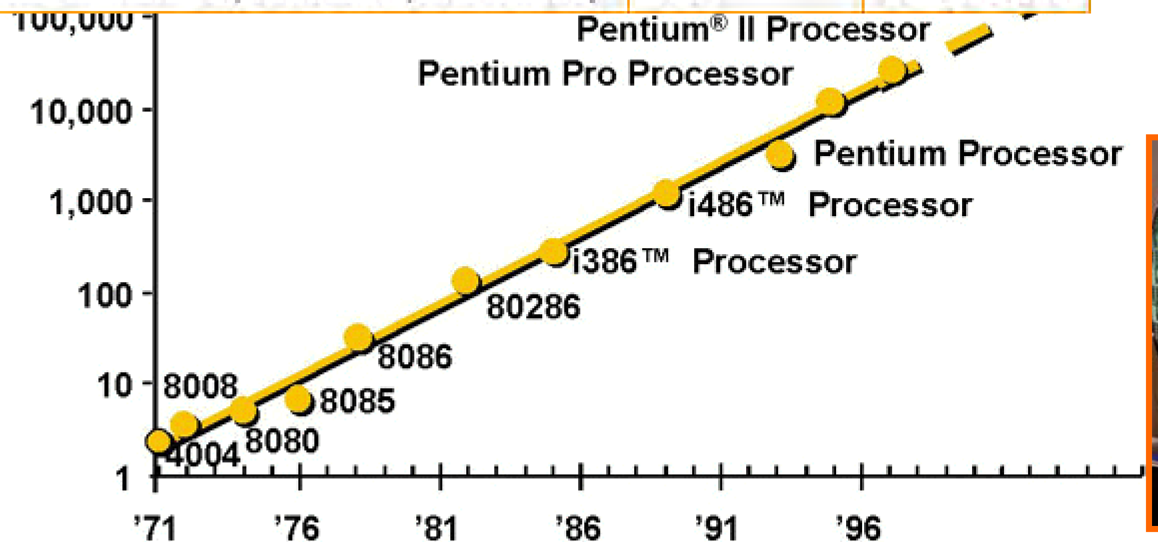
Gordon Moore (**Intel**) "Cramming more components onto Electronic Circuits", *Electronics Magazine* 19 April 1965

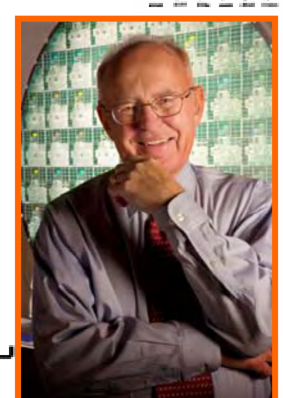
Unprecedented and expected to continue for 10-20 years.

Microprocessor	Year of Introduction	Transistors
4004	1971	2,300
8008	1972	2,500
8080	1974	4,500
8086	1978	29,000
Intel286	1982	134,000
Intel386™ processor	1985	275,000
Intel486™ processor	1989	1,200,000
Intel® Pentium® processor	1993	3,100,000
Intel® Pentium® II processor	1997	7,500,000
Intel® Pentium® III processor	1999	9,500,000
Intel® Pentium® 4 processor	2000	42,000,000
Intel® Itanium® processor	2001	25,000,000
Intel® Itanium® 2 processor	2003	220,000,000
Intel® Itanium® 2 processor (9MB cache)	2004	592,000,000

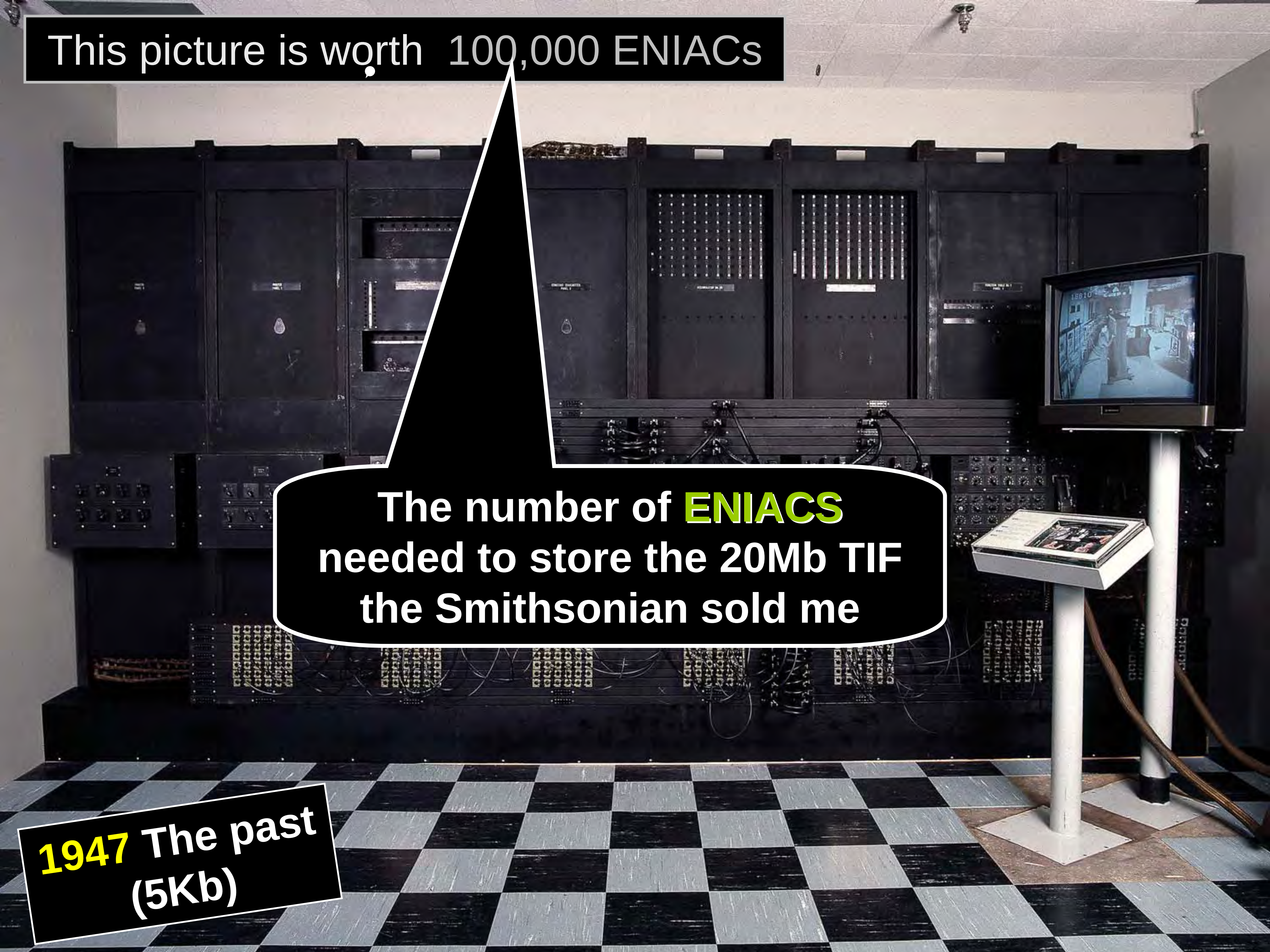






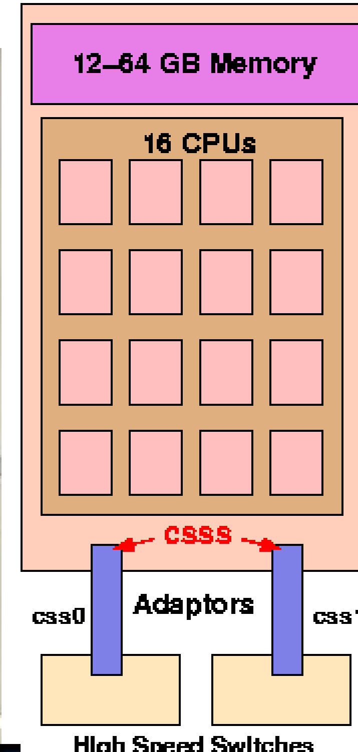


Moore's Law
1965 to 2005


This picture is worth 100,000 ENIACs
The number of ENIACS needed to store the 20Mb TIF the Smithsonian sold me
1947 The past (5Kb)

NERSC's 6000 cpu Seaborg in 2004 (10Tflops/sec)

- we need new software paradigms for `bigga-scale' hardware



The present

Mathematical Immersive Reality
in Vancouver

IBM BlueGene/L system at LLNL

System
(64 cabinets, 64x32x32)

Supercomputer doubles own record

2005

The Blue Gene/L supercomputer has broken its own record to achieve more than double the number of calculations it can do a second.

It reached 280.6 teraflops - that is 280.6 trillion calculations a second.

Blue Gene/L is the fastest computer in the world

5.6/11.2 GF/s
0.5 GB DDR

2.8/5.6 GF/s
4 MB

The future
2005-2010

2^{17} cpu's

Oct 2007 It has now run Linpack benchmark at over **596 Tflop /sec**
(5 x Canada)

"It says it's sick of doing things like inventories and payrolls, and it wants to make some break-throughs in astrophysics."

2. New Ways of Doing Math

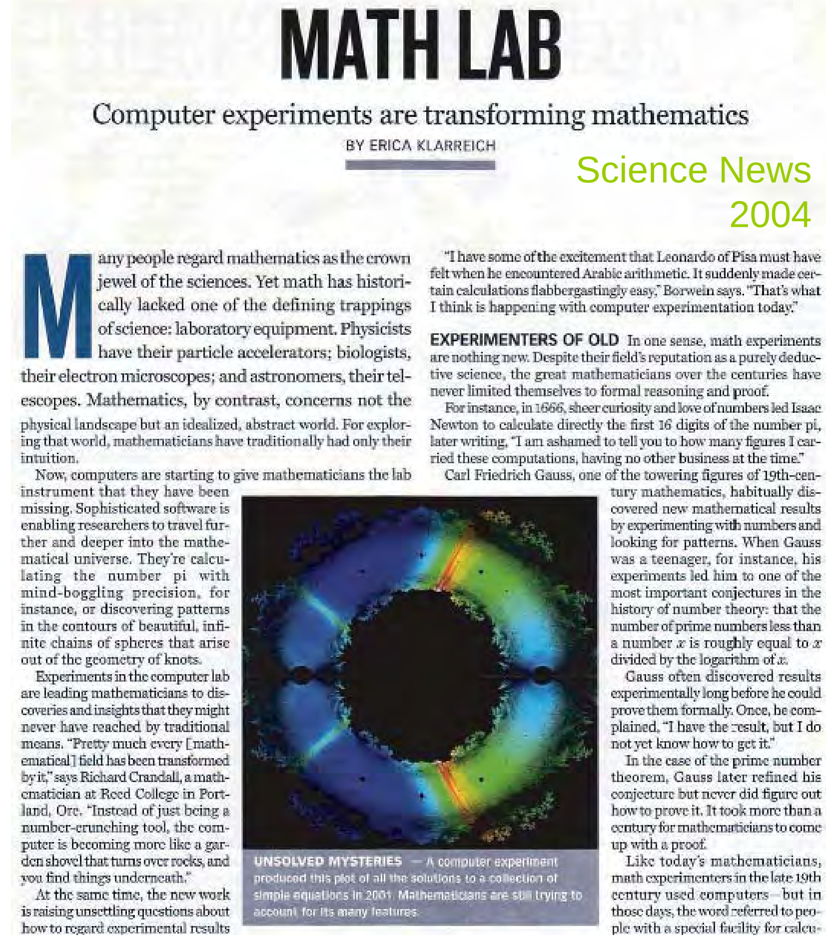
and related subjects: Computer Science, Statistics, Engineering, all Sciences, every other subject

- Experimentally on the Computer
- Visual or Haptic or Acoustic Output
- Simulations and Emersions
- With Web-services, Databases, Wikis, …

Also New Ways of Collaborating

Experimental Mathodology

1. Gaining insight and intuition
2. Discovering new relationships
3. Visualizing math principles
4. Testing and especially falsifying conjectures
5. Exploring a possible result to see if it merits formal proof
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results



MATH LAB

Computer experiments are transforming mathematics

BY ERICA KLARREICH

Many people regard mathematics as the crown jewel of the sciences. Yet math has historically lacked one of the defining trappings of science: laboratory equipment. Physicists have their particle accelerators; biologists, their electron microscopes; and astronomers, their telescopes. Mathematics, by contrast, concerns not the physical landscape but an idealized, abstract world. For exploring that world, mathematicians have traditionally had only their intuition.

Now, computers are starting to give mathematicians the lab instrument that they have been missing. Sophisticated software is enabling researchers to travel further and deeper into the mathematical universe. They're calculating the number pi with mind-boggling precision, for instance, or discovering patterns in the contours of beautiful, infinite chains of spheres that arise out of the geometry of knots.

Experiments in the computer lab are leading mathematicians to discoveries and insights that they might never have reached by traditional means. "Pretty much every [mathematical] field has been transformed by it," says Richard Crandall, a mathematician at Reed College in Portland, Ore. "Instead of just being a number-crunching tool, the computer is becoming more like a garden shovel that turns over rocks, and you find things underneath."

At the same time, the new work is raising unsettling questions about how to regard experimental results

UNSOLVED MYSTERIES — A computer experiment produced this plot of all the solutions to a collection of simple equations in 2001. Mathematicians are still trying to account for its many features.

"I have some of the excitement that Leonardo of Pisa must have felt when he encountered Arabic arithmetic. It suddenly made certain calculations flabbergastingly easy," Borwein says. "That's what I think is happening with computer experimentation today."

EXPERIMENTERS OF OLD In one sense, math experiments are nothing new. Despite their field's reputation as a purely deductive science, the great mathematicians over the centuries have never limited themselves to formal reasoning and proof.

For instance, in 1666, sheer curiosity and love of numbers led Isaac Newton to calculate directly the first 16 digits of the number pi, later writing, "I am ashamed to tell you to how many figures I carried these computations, having no other business at the time."

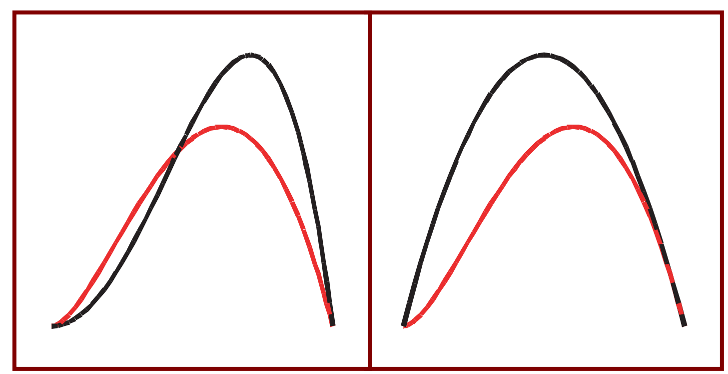
Carl Friedrich Gauss, one of the towering figures of 19th-century mathematics, habitually discovered new mathematical results by experimenting with numbers and looking for patterns. When Gauss was a teenager, for instance, his experiments led him to one of the most important conjectures in the history of number theory: that the number of prime numbers less than a number x is roughly equal to x divided by the logarithm of x.

Gauss often discovered results experimentally long before he could prove them formally. Once, he complained, "I have the result, but I do not yet know how to get it."

In the case of the prime number theorem, Gauss later refined his conjecture but never did figure out how to prove it. It took more than a century for mathematicians to come up with a proof.

Like today's mathematicians, math experimenters in the late 19th century used computers—but in those days, the word referred to people with a special facility for calcu-

Science News 2004



Comparing $-y^2\ln(y)$ (red) to $y-y^2$ and y^2-y^4

Experimental Mathematics in Action

David H. Bailey
Jonathan M. Borwein
Neil J. Calkin
Roland Girgensohn
D. Russell Luke
Victor H. Moll

The last twenty years have been witness to a fundamental shift in the way mathematics is practiced. With the continued advance of computing power and accessibility, the view that "real mathematicians don't compute" no longer has any traction for a newer generation of mathematicians that can really take advantage of computer-aided research, especially given the scope and availability of modern computational packages such as Maple, Mathematica, and MATLAB. The authors provide a coherent variety of accessible examples of modern mathematics subjects in which intelligent computing plays a significant role.

Advance Praise for *Experimental Mathematics in Action*

"Experimental mathematics has not only come of age but is quickly maturing, as this book shows so clearly. The authors display a vast range of mathematical understanding and connection while at the same time delineating various ways in which experimental mathematics is and can be undertaken, with startling effect."

—Prof. John Mason, Open University and University of Oxford

"Computing is to mathematics as telescope is to astronomy: it might not explain things, but it certainly shows 'what's out there.' The authors are expert in the discovery of new mathematical 'planets,' and this book is a beautifully written exposé of their values, their methods, their subject, and their enthusiasm about it. A must read."

—Prof. Herbert S. Wilf, author of *generatingfunctionology*

"From within the ideological blizzard of the young field of Experimental Mathematics comes this tremendous, clarifying book. The authors—all experts—convey this complex new subject in the best way possible; namely, by fine example. Let me put it this way: Discovering this book is akin to finding an emerald in a snowdrift."

—Richard E. Crandall, Apple Distinguished Scientist, Apple, Inc.

A K Peters, Ltd.

ISBN 978-1-56881-271-7

BAILEY
BORWEIN
CALKIN
GIRGENSOHN
LUKE
MOLL

Experimental Mathematics in Action

AK PETERS

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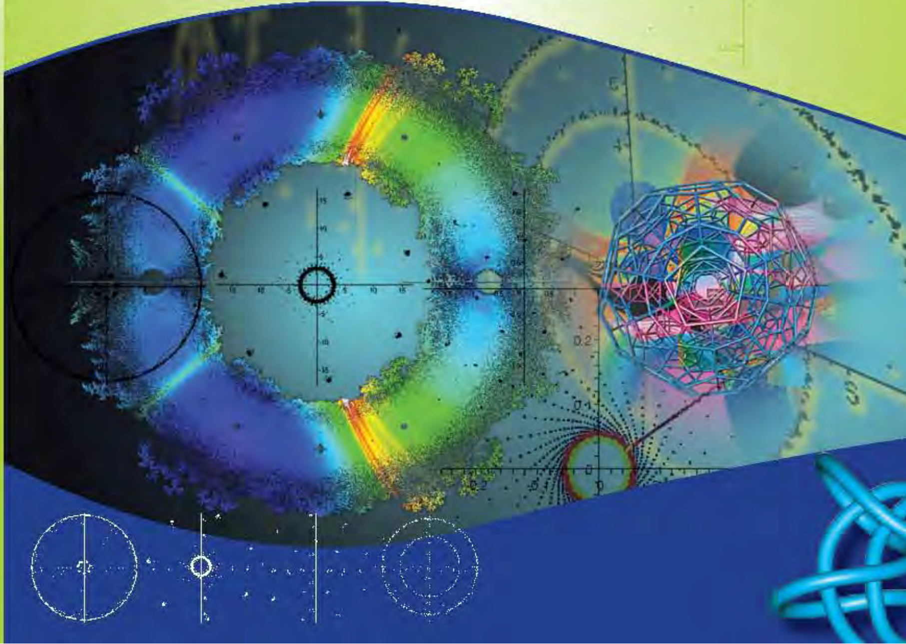
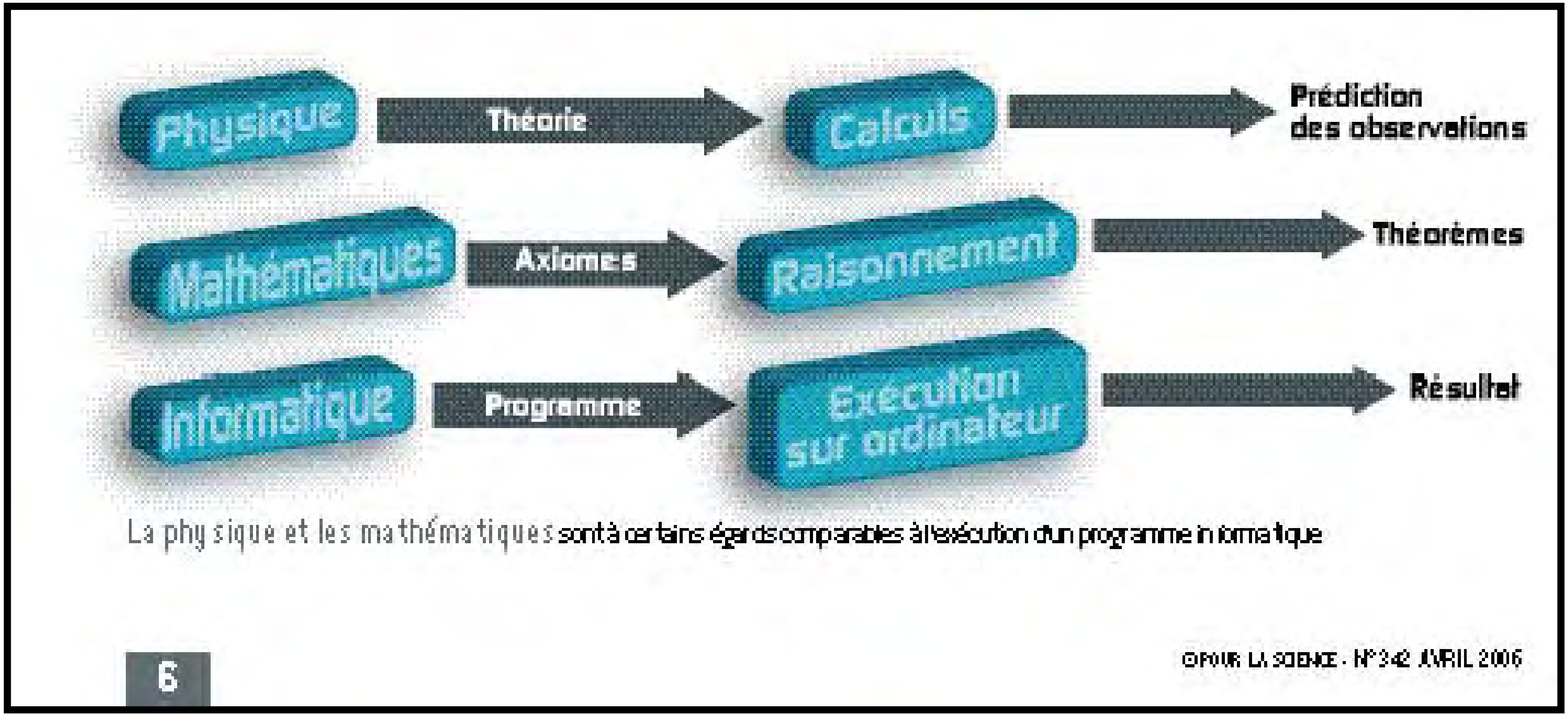
Much more use of visualization

Experimental Mathematics in Action



Math-Physics-Computing

- En français



How-To Training Sessions

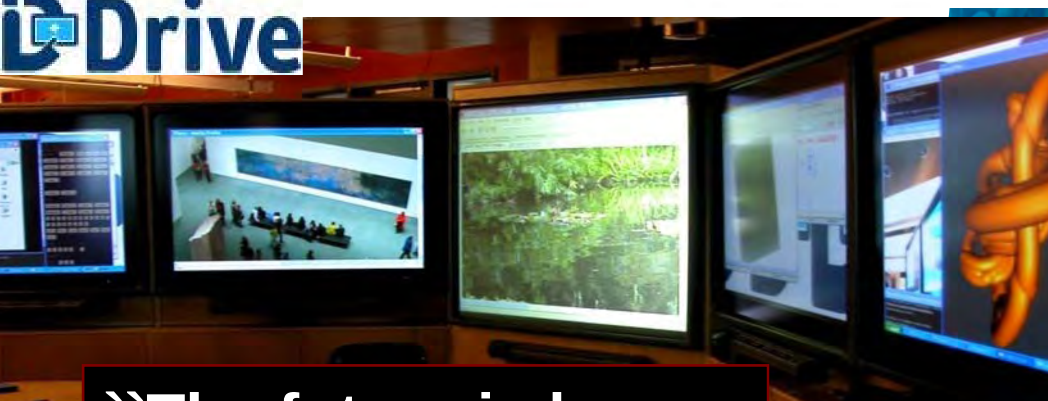
WestGrid

www.westgrid.ca

Brought to you using Access Grid technology

For more information contact Jana at 210-5489 or jana@netera.ca

D-Drive



``The future is here… **(William Gibson)** **… just not uniformly”**

Remote Visualization via **Access Grid**

- The touch sensitive interactive **D-DRIVE**
- Immersion & **Haptics**
- and the 3D .`**GeoWall**`

Haptics and Light Paths

D-DRIVE Doug our haptic mascot

Drive
Dalhousie Distributed Research Institute and Virtual Environment

Haptic Devices extend the world of I/O into the tangible and tactile

To test latency issues …

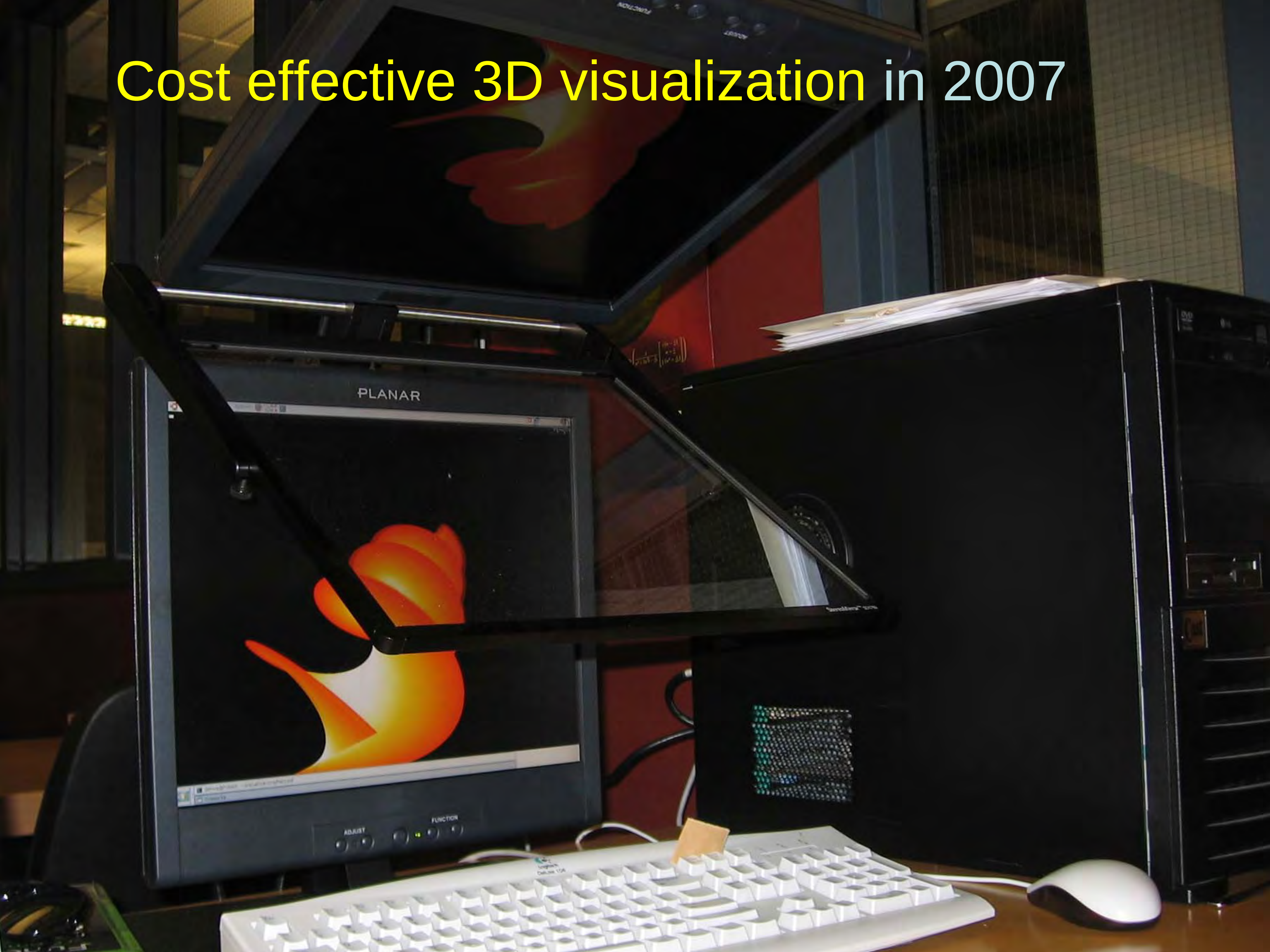
Sensable's **Phantom Omni**

We link multiple devices so two or more users may interact at a distance (**BC/NS Demo** April 06)

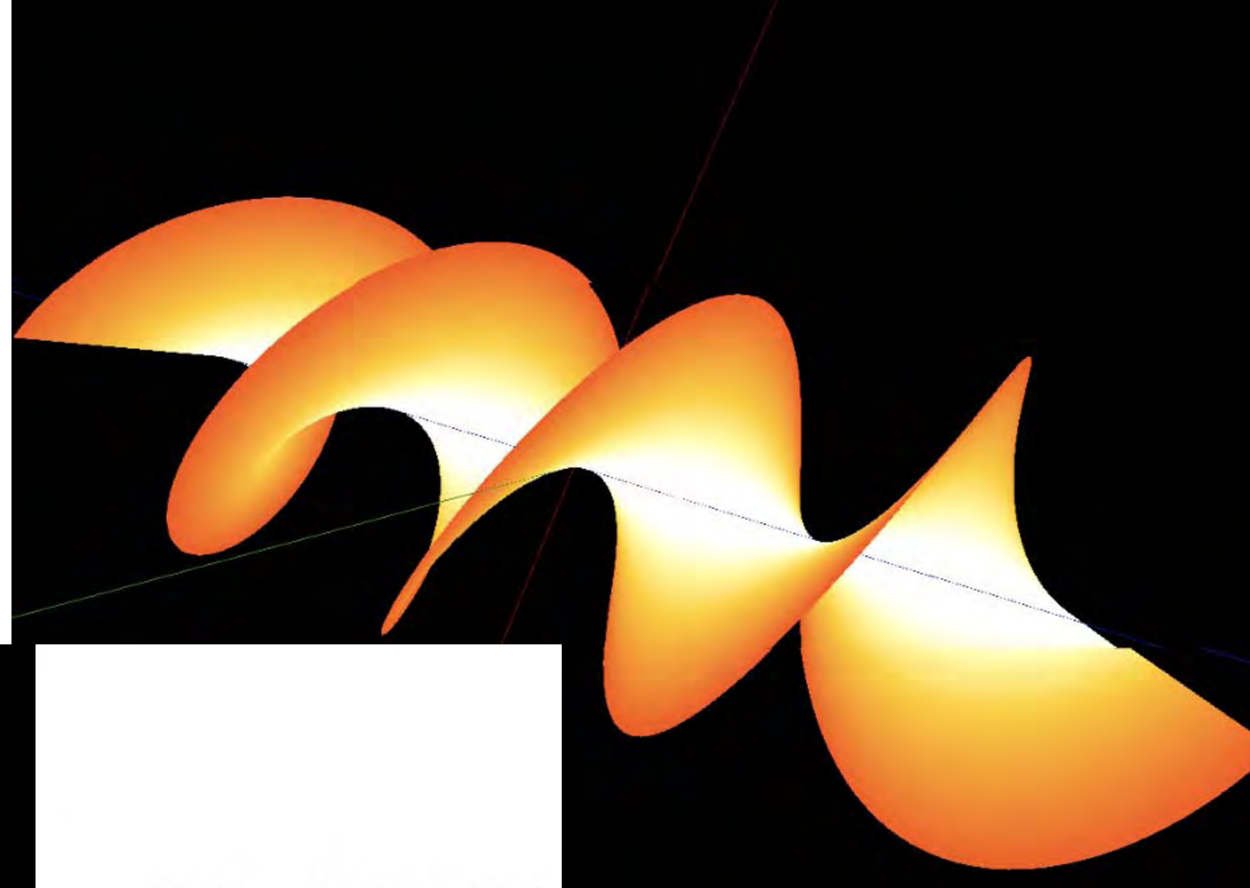
- in Museums, **Aware Homes**, elsewhere
- Kinesiology, Surgery, Music, Art …


Caveman Geometry
(2001)
Very cool for the one person with control

Cost effective 3D visualization in 2007



19th C model recent photo and 21st C rendition



Mathematical Form 0001

Helicoid: minimal surface.

$$x = a \sinh v \cos u$$
$$y = a \sinh v \sin u$$
$$z = au$$
$$(0 \le u < 2\pi, -\infty < v < \infty)$$

19th C Plaster Model
Kline and Schwartz

Hiroshi Sugimoto


Mathematical Form 0003

z

(0≤ u

Coast to Coast Seminar Series ('C2C')

AG Prototype for Atlantic Shared Curriculum Initiative (ASCI)

Lead partners:

Dalhousie D-Drive – Halifax Nova Scotia

IRMACS – Burnaby, British Columbia

Other Participants so far incude:

University of British Columbia, University of Alberta, University of Alberta, University of Saskatchewan, Lethbridge University, Acadia University, MUN, Mt Allison, St Francis Xavier University, University of Western Michigan, MathResources Inc, University of North Carolina

Tuesdays 3:30 – 4:30 pm Atlantic Time

✓http://projects.cs.dal.ca/ddrive/

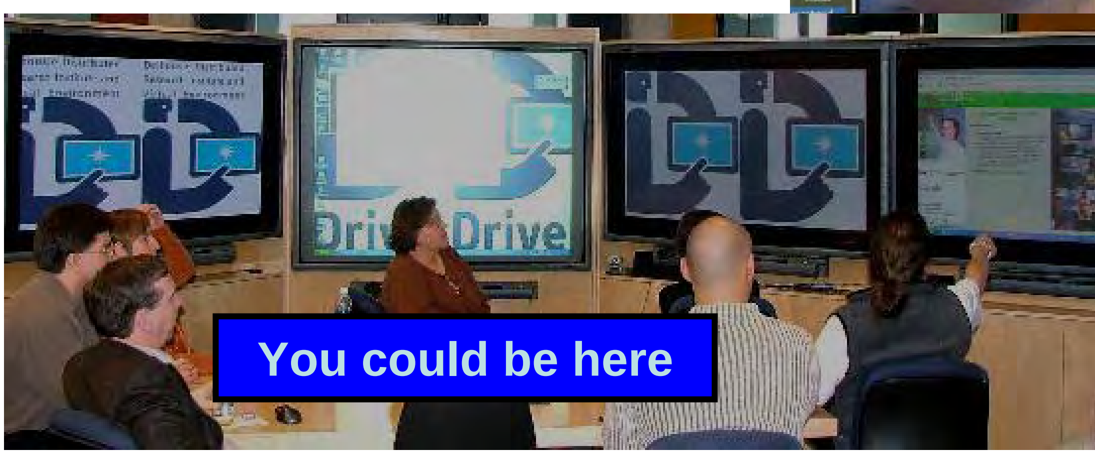
also a forthcoming book chapter

The Experience

Fully Interactive multi-way audio and visual interaction

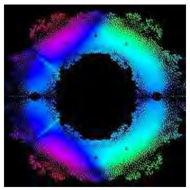
Given good bandwidth audio is much harder

The closest thing to being in the same room



Shared Desktop for viewing presentations or sharing software

Dalhousie Distributed Research Institute and Virtual Environment



Jonathan Borwein, Dalhousie University
Mathematical Visualization

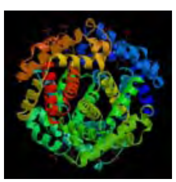
High Quality Presentations

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems

Peter Borwein, IRMACS
The Riemann Hypothesis

"No one explains chalk"

Jonathan Schaeffer, University of Alberta
Solving Checkers



Arvind Gupta, MITACS
The Protein Folding Problem

Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants

Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes

Future Libraries will include very complex objects

"Solving Checkers"

Speaker in Edmonton

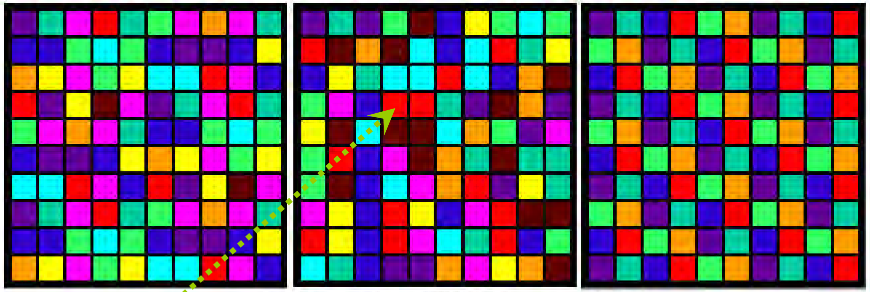
Audience in Vancouver

3. New Ways of Seeing Math

- The Colour Calculator
  - numbers as pictures
- The Inverse Calculator
  - numbers go in and symbols come out
- The Top Ten Numbers Website

- http://ddrive.cs.dal.ca/~isc/portal

A Colour and an Inverse Calculator (1995 & 2007)

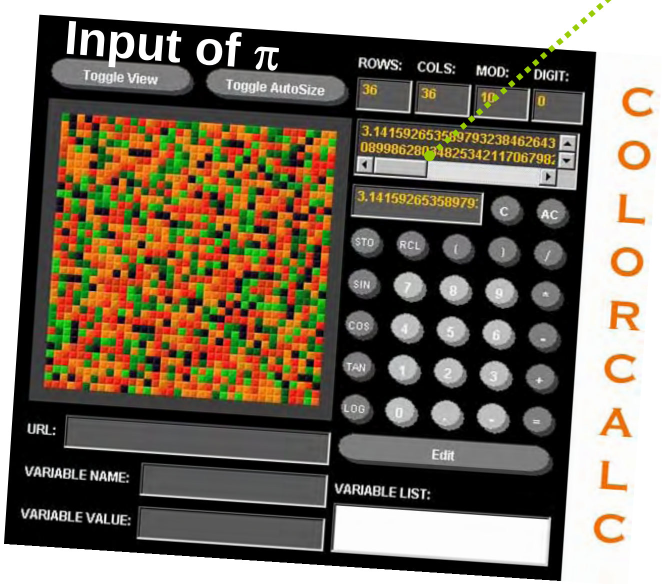


Archimedes: $223/71 < \pi < 22/7$

Inverse Symbolic Computation

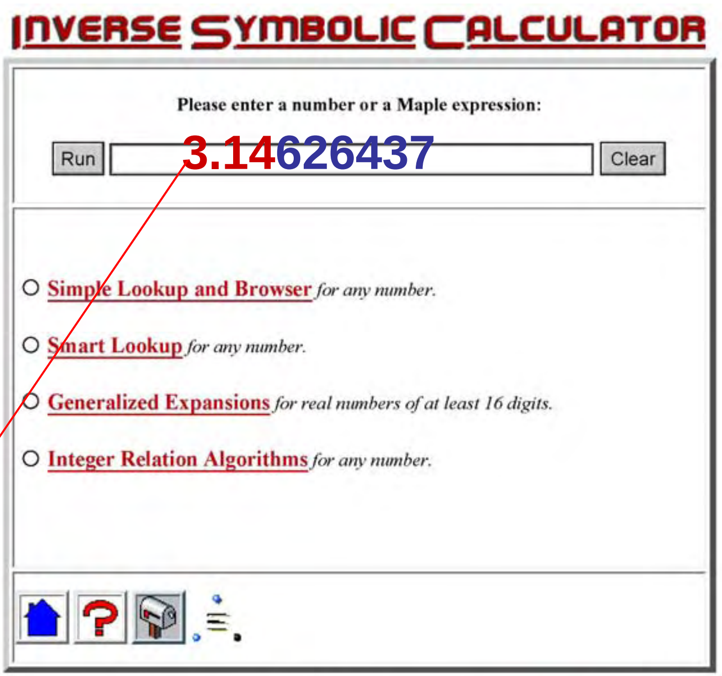
Inferring mathematical structure from numerical data

- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs ***micro-parallelism***
- It faces the "curse of exponentiality"
- Implemented as **identify** in Maple



identify(sqrt(2.)+sqrt(3.))

$$\sqrt{2}+\sqrt{3}$$



Expressions that are **not** numeric like ln(Pi*sqrt(2)) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.

relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

iSC+ inverse symbolic calculator

Standard lookup results for **12.587886229548403854**

exp(1)+Pi^2

ISC The original ISC

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

5.859874482 Try it!

3.146264370 Try it!

19.99909998 Try it!

ISC The original ISC

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

as input. However, for Maple syntax requiring too long for evaluation, a timeout has been implemented.

Visit

Jon Borwein's Webpage

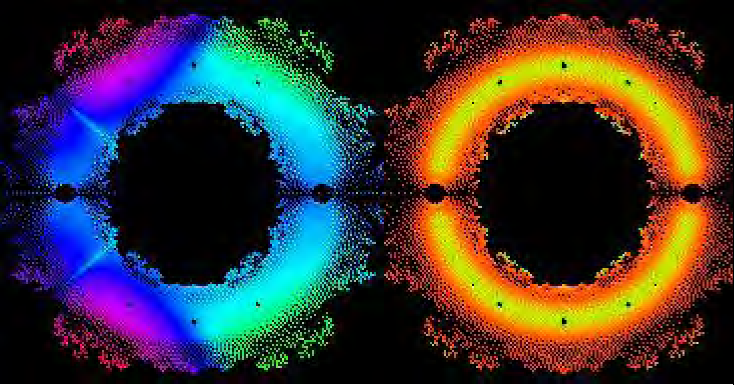
David Bailey's Webpage

Math Resources Portal

Webpage

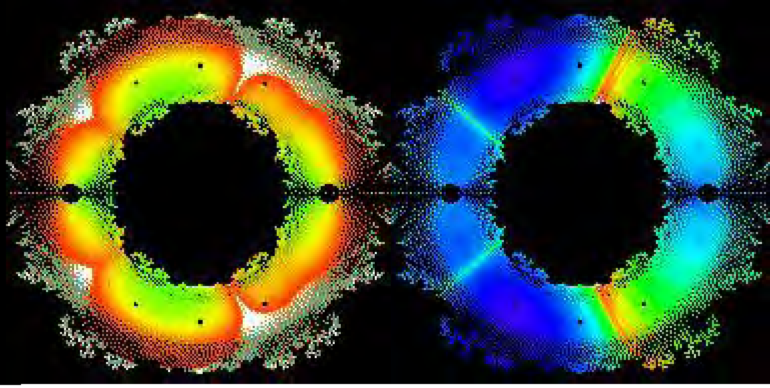
Math Resources Portal

- ISC+ runs on Glooscap
- Less lookup & more algorithms than 1995



Roots of Zeros

What you draw is what you see ("visible structures in number theory")



Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

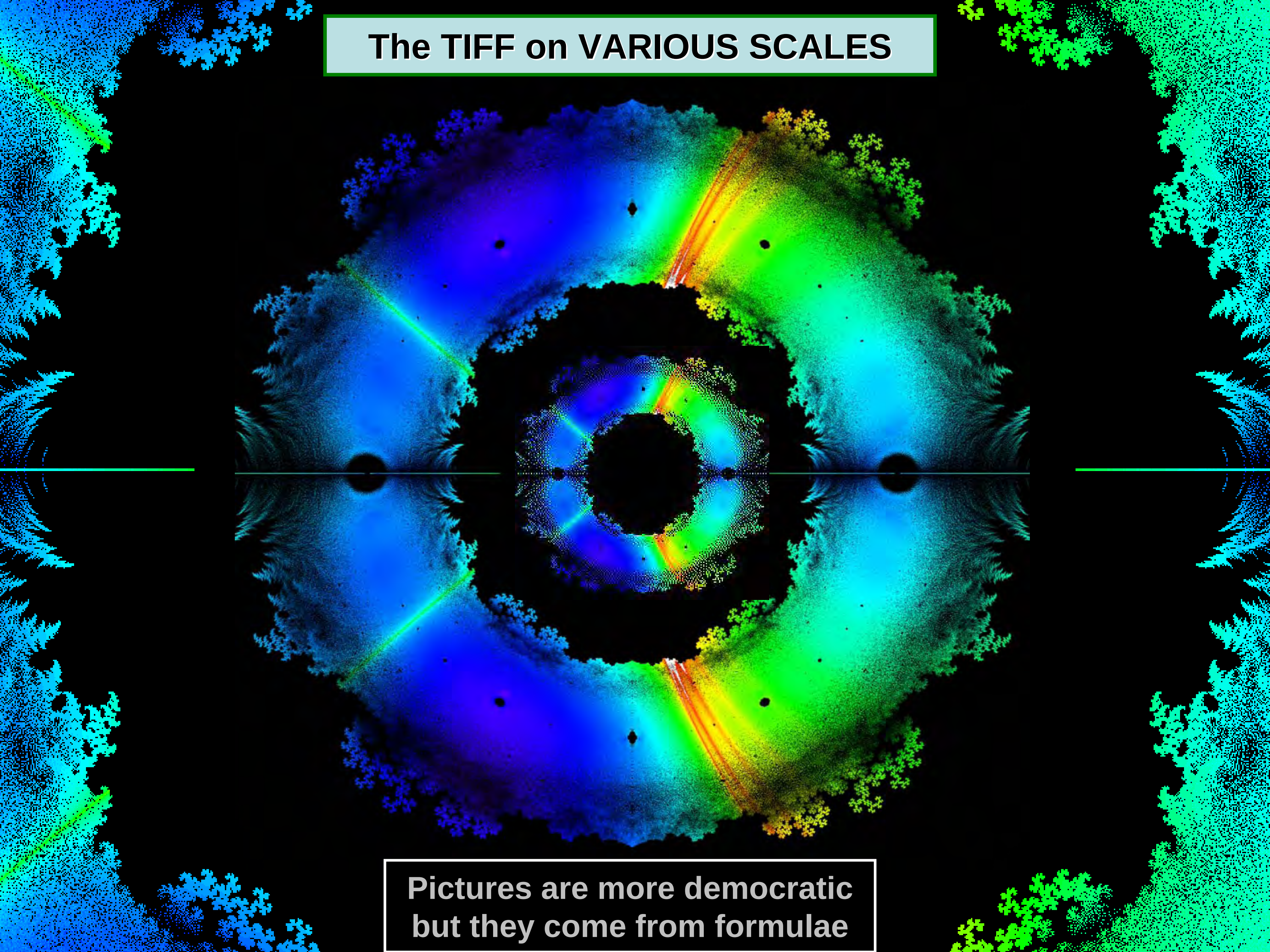
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. **The color scale represents a normalized sensitivity** to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at **3600 dpi**)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $\mathbf{x^9}$ term
- **The white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!" Greg Chaitin, Interview, 2000.

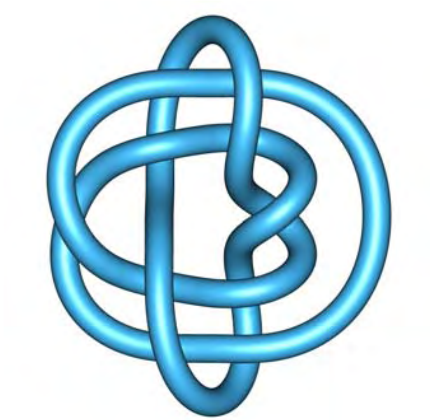
The TIFF on VARIOUS SCALES

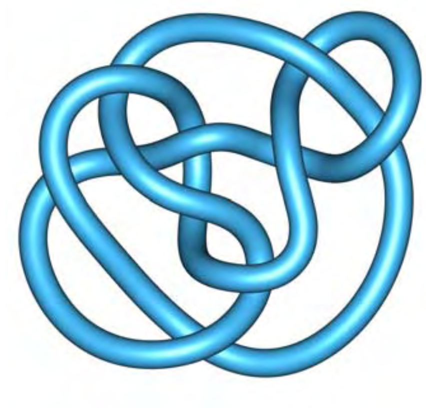


Pictures are more democratic but they come from formulae

The Perko Pair 10_{161} and 10_{162}

are two adjacent 10-crossing knots (1900)





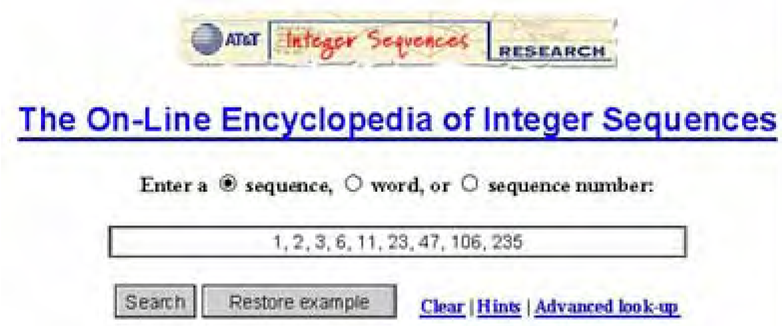
- **first shown to be the same by Ken Perko in 1974**
- **and beautifully made dynamic in KnotPlot (open source)**

"What it comes down to is our software is too hard and our hardware is too soft."

4. Amazing New Web Services

- Online Encyclopedia of Sequences

What is 1,2,3,6,11,23,47,106,235,…?



- Digital Library of Math Functions

What is an Airy Function?

Supernumerary Rainbow over Newton's birthplace

Soon the texts will also do the mathematics

PlanetMath.org

Wolfram MathWorld
the web's most extensive mathematics resource

Greetings from the On-Line Encyclopedia of Integer Sequences!

Matches (up to a limit of 30) found for 1 2 3 6 11 23 47 106 235 :
[It may take a few minutes to search the whole database, depending on how many matches are found (the second and later look
are faster)]

An Exemplary Database

ID Number: A000055 (Formerly M0791 and N0299)
URL: http://www.research.att.com/projects/OEIS?Anum=A000055
Sequence: 1,1,1,1,2,3,6,11,23,47,106,235,551,1301,3159,7741,19320,
48629,123867,317955,823065,2144505,5623756,14828074,
39299897,104636890,279793450,751065460,2023443032,
5469566585,14830871802,40330829030,109972410221
Name: Number of trees with n unlabeled nodes.
Comments: Also, number of unlabeled 2-gonal 2-trees with n 2-gons.
References F. Bergeron, G. Labelle and P. Leroux, Combinatorial Species and Tree-Like Structures, Camb. 1998, p. 279.
N. L. Biggs et al., Graph Theory 1736-1936, Oxford, 1976, p. 49.
S. R. Finch, Mathematical Constants, Cambridge, 2003, pp. 295-316.
D. D. Grant, The stability index of graphs, pp. 29-52 of Combinatorial Mathematics (Proceedings 2nd Australian Conf.), Lect. Notes Math. 403, 1974.
F. Harary, Graph Theory. Addison-Wesley, Reading, MA, 1969, p. 232.
F. Harary and E. M. Palmer, Graphical Enumeration, Academic Press, NY, 1973, p. 58 and 244.
D. E. Knuth, Fundamental Algorithms, 3d Ed. 1997, pp. 386-88.
R. C. Read and R. J. Wilson, An Atlas of Graphs, Oxford, 1998.
J. Riordan, An Introduction to Combinatorial Analysis, Wiley, 1958, p. 138.

Integrated real time use

- moderated
- 135,000 entries
- grows daily
- AP book had 5,000

Links: P. J. Cameron, Sequences realized by oligomorphic permutation groups J. Integ. Seqs. Vo
Steven Finch, Otter's Tree Enumeration Constants
E. M. Rains and N. J. A. Sloane, On Cayley's Enumeration of Alkanes (or 4-Valent Trees),
N. J. A. Sloane, Illustration of initial terms
E. W. Weisstein, Link to a section of The World of Mathematics.
Index entries for sequences related to trees
Index entries for "core" sequences
G. Labelle, C. Lamathe and P. Leroux, Labeled and unlabeled enumeration of k-gonal 2-tr

Formula: G.f.: A(x) = 1 + T(x)-T^2(x)/2+T(x^2)/2, where T(x) = x + x^2 + 2*x^3 + ...

http://dlmf.nist.gov First 21C database

Digital Library of Mathematical Functions

Chapter AI. Airy & Related Functions

Properties

Index
Notation
Search

for what Go

Show Annotations

Need Help?

About the Project

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DLMF_feedback@nist.gov

January 12, 2006

Supported by NIST National Institute of Standards and Technology and NSF National Science Foundation Celebrating 50 Years

§AI.3. Graphs & Visualizations | §AI.5. Integral Representations

§AI.4. Maclaurin Series

For $z \in \mathbb{C}$

AI.4.1

$$\mathrm{Ai}(z) = \mathrm{Ai}(0)\left(1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\cdots\right)+\mathrm{Ai}'(0)\left(z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\cdots\right),$$

Symbols used:
AiryAi, cdots and z

A&S Ref:
10.4.2 (with 10.4.4 and 10.4.5)

Encodings:
LaTeX

Parsed:

- AiryAi@(z) = AiryAi@(0) * (1 + (1 / 3!) * z ^ 3 + ((1 cdot 4) / 6!) * z ^ 6 + ((1 cdot 4 cdot 7) / 9!) * z ^ 9 + cdots) + (diffop@(AiryAi, 1))@(0) * (z + (2 / 4!) * z ^ 4 + ((2 cdot 5) / 7!) * z ^ 7 + ((2 cdot 5 cdot 8) / 10!) * z ^ 10 + cdots)

- **Formula level metadata**
- **Mathematical searching**
- **Accessible output**
- **Latex, PNG MathML**

AI.4.2

$$\mathrm{Ai}'(z) = \mathrm{Ai}'(0)\left(1+\frac{2}{3!}z^3+\frac{2\cdot 5}{6!}z^6+\frac{2\cdot 5\cdot 8}{9!}z^9+\cdots\right)+\mathrm{Ai}(0)\left(\frac{1}{2!}z^2+\frac{1\cdot 4}{5!}z^5+\frac{1\cdot 4\cdot 7}{8!}z^8+\cdots\right),$$

AI.4.3

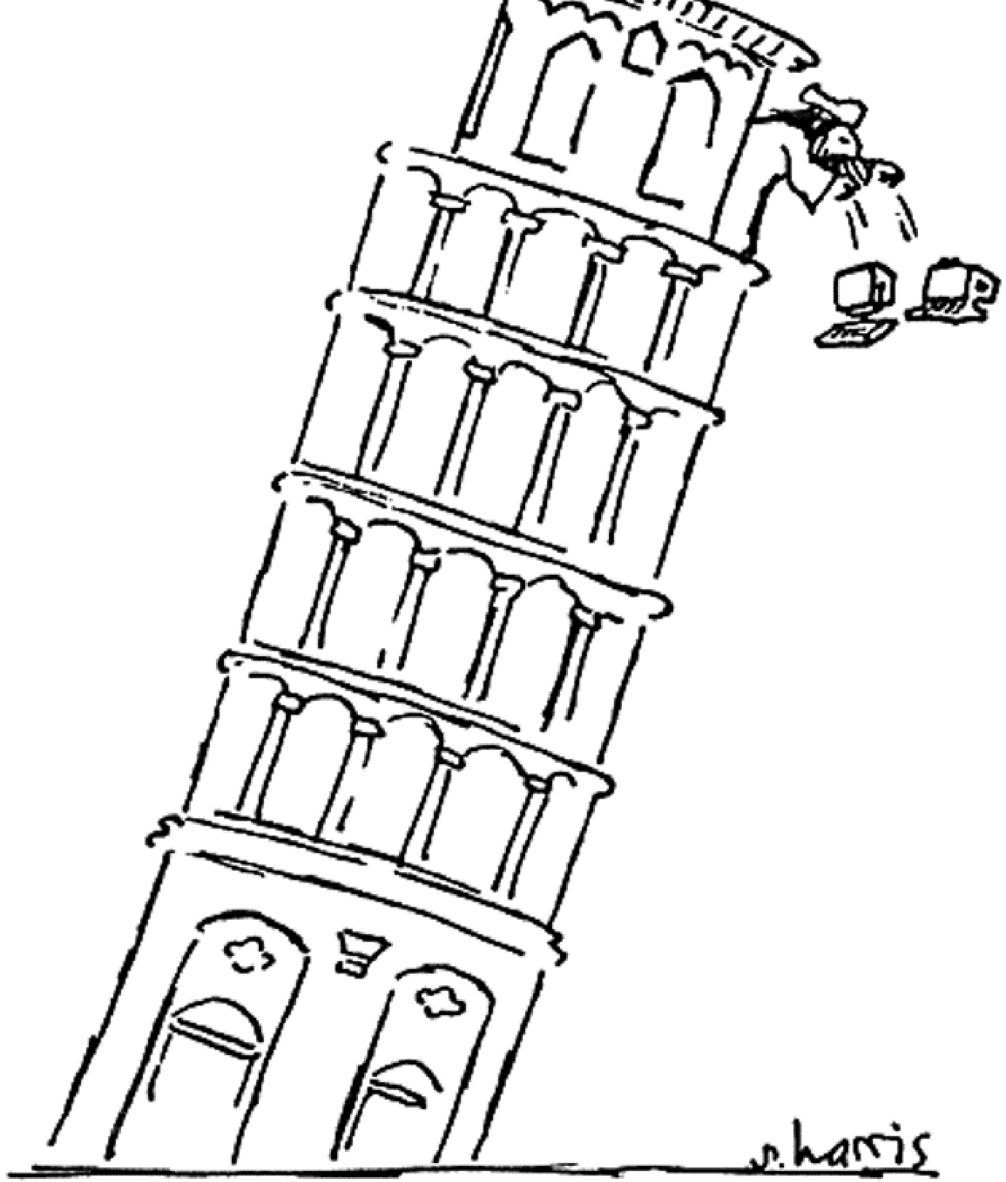
$$\mathrm{Bi}(z) = \mathrm{Bi}(0)\left(1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\cdots\right)+\mathrm{Bi}'(0)\left(z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\cdots\right),$$

AI.4.4

$$\mathrm{Bi}'(z) = \mathrm{Bi}'(0)\left(1+\frac{2}{3!}z^3+\frac{2\cdot 5}{6!}z^6+\frac{2\cdot 5\cdot 8}{9!}z^9+\cdots\right)+\mathrm{Bi}(0)\left(\frac{1}{2!}z^2+\frac{1\cdot 4}{5!}z^5+\frac{1\cdot 4\cdot 7}{8!}z^8+\cdots\right).$$

§AI.3. Graphs & Visualizations | §AI.5. Integral Representations

The faint line below the main colored arc is a `supernumerary rainbow', produced by the interference of different sun-rays traversing a raindrop and emerging in the same direction. For each color, the intensity profile across the rainbow is an Airy function. Airy invented his function in 1838 precisely to describe this phenomenon more accurately than Young had done in 1800 when pointing out that supernumerary rainbows require the wave theory of light and are impossible to explain with Newton's picture of light as a stream of independent corpuscles. The house in the picture is Newton's birthplace.



IF THERE WERE COMPUTERS IN GALILEO'S TIME

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Enigma

J.M. Borwein and D.H. Bailey, *Mathematics by Experiment: Plausible Reasoning in the 21st Century* A.K. Peters, 2003. and R. Girgensohn, *Experimentation in Mathematics: Computational Paths to Discovery,* A.K. Peters, 2004, **2008**.
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D.H. Bailey and J.M Borwein, "Experimental Mathematics: Examples, Methods and Implications," *Notices Amer. Math. Soc.*, **52** No. 5 (2005), 502-514.

J. Borwein, D. Bailey, N. Calkin, R. Girgensohn, R. Luke, and V. Moll, *Experimental Mathematics in Action*, A.K. Peters, 2007.

"The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

- J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.