Locally pro-p contraction groups are nilpotent

George A. Willis & H. Glöckner

August 20th, 2018

Abstract

A *contraction group* is a pair (G, α) in which G is a locally compact group and α is an automorphism of G such that $\alpha^n(x) \to 1$ as $n \to \infty$.

In joint work with H. Glöckner, it is shown that every contraction group is the direct sum of closed subgroups

$$G = D \oplus T$$

with D divisible (*i.e.* for every $x \in D$ and $n > 0$ there is $y \in D$ with $y^n = x$) and T torsion (*i.e.*, there is $n > 0$ such that $x^n = 1$ for every $x \in T$). Furthermore, D is the direct sum

$$D = \bigoplus_{i=1}^{k} D_{p_i}$$

of p_i-adic analytic nilpotent contraction groups for some prime numbers $p_1, \ldots, p_k$.

The torsion subgroup T may also be written as a composition series of simple contraction groups. In the case when all the composition factors are of the form $(\mathbb{F}_p(\!(t)\!), \alpha)$ with α being the automorphism of multiplication by p, it follows easily that G is a solvable group. These ideas will be explained in the talk and a sketch will be presented of a proof that G is in fact nilpotent in this case.